Income Tax: Enforcement and Compliance

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Lecture 1

Income Tax I

Let

- y denote the income level of an individual
- e denote the income evaded/hidden from tax authority
- $y - e$ denote the income reported for tax purpose
- t the tax rate
- π probability that tax authority will detect and fine the evasion
- $F(e, t)$ fine in case of detection; $F(0) = 0$, $F'(e) > 0$
- Fine is paid over and above the tax on hidden income - you should also consider the case when fine is on hidden income.
- There is only one period and all of disposable income is consumed within the period

Tax evasion and payoff I

Let

- Fine be imposed on tax evaded, *et*,
- $F(e,t) = f(e)et$

Consumption levels:

ase 1: No evasion : $c_0 = y(1-t)$

ase 2 : When evasion goes undetected (with probability $(1-\pi)$:
$\bar{c} = y - (y-e)t = y(1-t) + et$

ase 3 : When Evasion is detected (with probability (π):
$\underline{c} = y - (y-e)t - (1+f(e))et = y(1-t) - f(e)et$

Hence,the rate of return on *tax evaded*, *et*, given by

$$\left[\begin{array}{ccc} -f(e) & \text{with} & \pi \\ 1 & \text{with} & 1-\pi \end{array} \right.$$

Risk-neutral taxpayers I

Assume

- tax payers to be risk-neutral

Tax-payer's Expected Utility maximization problem is given as:

$$\max_{e}\{\pi u(\underline{c}) + (1-\pi)u(\bar{c})\},$$

$$\max_{e}\{\pi(y(1-t) - f(e)et) + (1-\pi)(y(1-t) + et)\},$$

probability π is exogenously given. So the FOC is given by

$$\pi[-f(e).t - t.e.f'(e)] + (1-\pi).t = 0 \tag{1}$$

If $f'(e) = 0$, then there will be a corner solution - either no evasion or full evasion. When $f'(e) > 0$, then evasion level is

$$e^*(t,\pi,y) = \frac{(1-\pi) - \pi f(e)}{\pi f'(e)}$$

Risk-aversion and Compliance: Illustration I

Assume

- tax payers to be risk-averse
- to get tax evasion assume: $1 - \pi(1 + f(e = 0) > 0$
- Linear Fine Function: $F(et) = fet$
- Utility function: $u(c) = \sqrt{c}$

$$EU = \pi[\sqrt{(1-t)y - fet}] + (1-\pi)[\sqrt{(1-t)y + et}$$

FOC w.r.t. e is given by

$$\phi(e, t, \pi, y) = -\frac{f\pi}{\sqrt{\underline{c}}} + (1-\pi)\frac{1}{\sqrt{\bar{c}}} = 0$$

Thus e^* solves:

$$\frac{1-\pi}{\pi} = \frac{f\sqrt{\bar{c}}}{\sqrt{\underline{c}}} = \frac{f\sqrt{(1-t)y + et}}{\sqrt{(1-t)y - fet}}$$

Risk-aversion and Compliance: Illustration II

Proposition

$\frac{de^*}{d\pi} < 0$, $\frac{de^*}{df} < 0$. *What about* $\frac{de^*}{dt}$ *?*

For given t, π and y, the above FOC can be written as:

$$\phi(e, t, \pi, y) = 0$$

Let $e^*(t, \pi, y)$ solve the FOC. Note: For $\tau = t, \pi, y$,

$$\frac{\partial \phi}{\partial \tau} + \frac{\partial \phi}{\partial e^*}\frac{\partial e^*}{\partial \tau} = 0 \qquad (2)$$

So

$$\frac{\partial e^*}{\partial \tau} = -\frac{\frac{\partial \phi}{\partial \tau}}{\frac{\partial \phi}{\partial e^*}}$$

Here we have $\frac{\partial \phi}{\partial e^*} < 0$. Moreover,

Risk-aversion and Compliance: Illustration III

$\frac{\partial \phi}{\partial \pi} < 0$ and $\frac{\partial \phi}{\partial f} < 0$ and $\frac{\partial \phi}{\partial t} < 0$ and $\frac{\partial \phi}{\partial y} > 0$ always.

So, we get:

$$\frac{\partial e^*}{\partial \pi} < 0 \;\; and \;\; \frac{\partial e^*}{\partial f} < 0.$$

Similarly, we get

$$\frac{\partial e^*}{\partial y} > 0 \;\; and \;\; \frac{\partial e^*}{\partial t} < 0$$

Note : We can get the same signs by directly differentiating e^* with respect to different parameters.

Risk-aversion and (Non)Compliance Decision I

In general Tax-payer's Expected Utility maximization problem is given as:

$$\max_e \{\pi u(\underline{c}) + (1-\pi)u(\bar{c})\},$$

$$\max_e \{\pi u(y(1-t) - f(e)et) + (1-\pi)u(y(1-t)+et)\},$$

probability π is exogenously given. So the FOC is given by

$$\pi.u_{\underline{c}}(\underline{c})[-f(e).t - t.e.f'(e)] + (1-\pi).t.u_{\bar{c}}(\bar{c}) = 0 \tag{3}$$

$$e^*(t,\pi,y) = \frac{(1-\pi)u_{\bar{c}}(\bar{c}) - \pi.u_{\underline{c}}f(e)}{\pi.u_{\underline{c}}(\underline{c}).f'(e)}$$

Let

$$\phi(e,t,\pi,y) \equiv \pi.u_{\underline{c}}(\underline{c})[-f(e).t - t.e.f'(e)] + (1-\pi).t.u_{\bar{c}}(\bar{c})$$

So, for given t, π and y, FOC (1) can be written as:

$$\phi(e,t,\pi,y) = 0$$

Risk-aversion and (Non)Compliance Decision II

Let $e^*(t, \pi, y)$ solve the FOC. Note:

$$\frac{\partial \phi}{\partial \pi} + \frac{\partial \phi}{\partial e^*}\frac{\partial e^*}{\partial \pi} = 0 \tag{4}$$

So

$$\frac{\partial e^*}{\partial \pi} = -\frac{\frac{\partial \phi}{\partial \pi}}{\frac{\partial \phi}{\partial e^*}}$$

$$\begin{aligned}
\frac{\partial \phi}{\partial e} &= -\pi[.u_{\underline{c}}(\underline{c})[2f'(e) + e.f''(e)] - [f(e) + e.f'(e)]^2.u_{\underline{cc}}(\underline{c})] \\
&+ (1-\pi).u_{\bar{c}\bar{c}}(\bar{c}) \\
\frac{\partial \phi}{\partial t} &= \pi[f(e) + e.f'(e)].u_{\underline{cc}}.(\underline{c})(f(e).e + y) - (1-\pi).u_{\bar{c}\bar{c}}(\bar{c})(y - e) \\
\frac{\partial \phi}{\partial \pi} &= -u_{\underline{c}}(\underline{c}).[f(e) + e.f'(e)] - u_{\bar{c}}(\bar{c}) \\
\frac{\partial \phi}{\partial y} &= (1-t)[-\pi(f(e) + ef'(e)).u_{\underline{cc}}.(\underline{c}) + (1-\pi)u_{\bar{c}\bar{c}}.(\bar{c})]
\end{aligned}$$

Risk-aversion and (Non)Compliance Decision III

$$\frac{\partial\phi}{de} < 0 if\ f''(e) \geq\ 0,\ \&\frac{\partial\phi}{\partial\pi} < 0\ \ \&\ \frac{\partial\phi}{\partial y} > 0$$

Note: $\frac{\partial\phi}{\partial t} < 0$ if

$$\pi[f(e) + e.f'(e)].u_{\underline{c}\underline{c}}.(\underline{c})(f(e).e + y) < (1 - \pi).u_{\bar{c}\bar{c}}(\bar{c})(y - e)$$

We know that $(y + f(e)e) > (y - e)$ and under Decreasing Absolute Risk Aversion

$$-\frac{u_{\underline{c}\underline{c}}(\underline{c})}{u_{\underline{c}}(\underline{c})} > -\frac{u_{\bar{c}\bar{c}}(\bar{c})}{u_{\bar{c}}(\bar{c})}.$$

That is,

$$\frac{(f(e).e + y)u_{\underline{c}\underline{c}}.(\underline{c})}{u_{\underline{c}}(\underline{c})} < \frac{(y - e)u_{\bar{c}\bar{c}}(\bar{c})}{u_{\bar{c}}(\bar{c})}$$

Risk-aversion and (Non)Compliance Decision IV

From FOC we have

$$\pi.u_{\underline{c}}(\underline{c})[-f(e).t - t.e.f'(e)] + (1-\pi).t.u_{\bar{c}}(\bar{c}) = 0, i.e.,$$

$$\frac{\pi[f(e) + e.f'(e)]}{1-\pi} = \frac{u_{\bar{c}}(\bar{c})}{u_{\underline{c}}(\underline{c})}$$

Therefore the condition for $\frac{\partial\phi}{\partial t} < 0$ will always hold.
From (2)

$$\frac{\partial e^*}{\partial \pi} = -\frac{\frac{\partial\phi}{\partial\pi}}{\frac{\partial\phi}{\partial e}} < 0$$

That is, Ceteris paribus, the level of evaded income e^* decreases as probability of detection (π) increases.

Risk-aversion and (Non)Compliance Decision V

Similarly, the effect of tax rate on the evaded income is negative.

$$\frac{\partial e^*}{\partial t} = -\frac{\frac{\partial \phi}{\partial t}}{\frac{\partial \phi}{\partial e}} < 0$$

Ceteris paribus, the effect of change in actual income on evaded income .

$$\frac{\partial e^*}{\partial y} = -\frac{\frac{\partial \phi}{\partial y}}{\frac{\partial \phi}{\partial e}} > 0$$

Quality of Governance and Compliance: Example I

- Tax revenue is given by $T = (y, e, t, \pi)$
- λ fraction of the tax collected is spent of public good - the rest is misappropriated by the government
- Public good production function is such that

$$g = g(\lambda, T) = g(\lambda, y, e, t, \pi); \quad g'(e) < 0$$

Example:

- Constant Fine $F() = fet$
- $g = \lambda(y - e)t$
- Utility function: $u(c, g) = \sqrt{c} + \sqrt{g}$

Quality of Governance and Compliance: Example II

$$EU = \pi[\sqrt{(1-t)y - fet} + \sqrt{g}] + (1-\pi)[\sqrt{(1-t)y + et} + \sqrt{g}$$

FOC is given by

$$\begin{aligned} (1-\pi)[u_{\bar{c}}(\bar{c}, g)t + u_g(\bar{c}, g)\lambda g'(e)] & \\ +\pi[u_{\underline{c}}(\underline{c}, g)[-ft] + u_g(\underline{c}, g)\lambda g'(e)] & = 0 \end{aligned}$$

$$\phi(e, t, \pi, y) = -\frac{f\pi}{\sqrt{\underline{c}}} + (1-\pi)\frac{1}{\sqrt{\bar{c}}} - \frac{\lambda t}{2\sqrt{g}} = 0$$

Quality of Governance and Compliance: Example III

Thus e^* solves:

$$\frac{1-\pi}{\pi} = \frac{f\sqrt{\bar{c}}}{\sqrt{\underline{c}}} + \frac{\lambda\sqrt{\bar{c}}}{\pi\sqrt{g}} = \frac{f\sqrt{(1-t)y+et}}{\sqrt{(1-t)y-fet}} + \frac{\lambda\sqrt{\bar{c}}}{\pi\sqrt{g}}$$

Now ,

$$e^* > e^{*p}$$

Question

$\frac{de^{*p}}{d\pi}$, $\frac{de^{*p}}{df}$ $\frac{de^{*p}}{dt}$, etc ?