202: Dynamic Macroeconomic Theory
Samuelson-Diamond Overlapping Generations Model

Mausumi Das

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Neoclassical Growth Revisited: Micro-founded Frameworks with Optimizing Agents

- You are familiar with the Solow model which provides the basic neoclassical framework for analysing issues pertaining to economic growth.

- Solow model of course assumes an exogenously given savings ratio, which implies that households *do not* decide on their optimal consumption/savings plan based on some explicit maximization exercise.

- The Solow model was later extended to allow for optimizing agents.

- There are two frameworks which allow for optimal consumption and savings behaviour by households:

  1. The Ramsey-Cass-Koopmans Inifinite Horizon Framework (henceforth R-C-K);
  2. The Samuelson-Diamond Overlapping Generations Framework (henceforth OLG).

- The R-C-K framework and the OLG framework have now become the two fundamental workhorses of modern macro analyses.

- The basic difference between the two is that in the R-C-K model agents optimize over infinite horizon; while in the OLG model, agents optimize over a finite time horizon (usually 2 periods).

- As we shall see, this apparently innocuous difference in terms of time horizon spells out very different growth trajectories for the two models.

Ramsey-Cass-Koopmans Inifinite Horizon Model: A Recap

- We have discussed the R-C-K model in detail in the core macro course and analysed its growth implications.

- We found that the growth implications of the R-C-K model are identical to that of the Solow model:
  - In the absence of technical progress, per capita income does not grow in the long run; aggregate output in the long run grows at the same rate as the (exogenous) rate of growth of population;
  - In the short/medium run (during transition) poorer economies grow faster than the richer ones (provided they have similar parameters).

- We also noted that the R-C-K model adequately addresses one major shortcoming of the Solow model: the unique non-trivial steady state in the R-C-K model is *always dynamically efficient*.

- Do these conclusions hold for the OLG framework as well - which also allows agents to choose their consumption/savings path based on explicit optimization? This is the question that we shall take up now.

Samuelson-Diamond Overlapping Generations Framework

- This framework was first developed by Samuelson (1958) in the context of an exchange economy, which was later extended to a production economy by Peter Diamond (1965).
- Each agent now lives exactly for T periods. For convenience, we shall assume that $T = 2$.
- We shall denote these two periods of an agent's life time by 'youth' and 'old-age' respectively.
- The agent works only in the first period of his life (when young) and is retired in the second period (when old).
- Thus he has to make provisions for his old-age consumption from his first period wage income itself (through savings).
- The agent optimally decides about his consumption profile by maximizing his life-time utility (to be specified later).

OLG Model: Production Side Story

- Once again, the production side story in the OLG model is identical to that of Solow.
- Thus the economy starts with a given stock of capital (K_t) and a given stock of labour force (N_t) at time t.
- Notice that since people do not work during their old age, the current labour force consist only of the current youth.
- We also assume that all firms have access to an identical production technology - **which satisfies all standard neoclassical properties**.
- The firm-specific production functions can be aggregated to generate an aggregate production function such that :

$$Y_t = F(K_t, N_t).$$

- At every point of time the market clearing wage rate and the rental rate of capital are given by:

$$w_t = F_N(K_t, N_t); \quad r_t = F_K(K_t, N_t).$$

OLG Model: Household Side Story

- There are H households or dynasties in the economy. In every household, at any point of time t, there are two cohorts of agents - those who are born in period t ('generation t' - who are currently young) and those who were born in the previous period ('generation $t-1$' - who are currently old): hence the name 'overlapping generations'.

- Thus at any point of time t, total population consists of two successive generations of people:

$$L_t = N_t + N_{t-1}.$$

(Notice that although total population is L_t, total labour force at time t in only N_t.)

- We shall assume that population in successive generations grows at a constant rate n :

$$N_{t+1} = (1+n)N_t.$$

- Hence total population in the economy (L_t) also grows at the same constant rate n.

Life Cycle of a Representative Member of Generation t:

- All agents **within** a generation are identical. So we can talk in terms of a representative agent belonging to 'generation t'.

- The agent is born at the beginning of period t with an endowment of one unit of labour.

- All agents in this model are selfish - they care only about their own consumption/utility and not about their childrens' utility. Hence they do not leave any bequest.

- No bequest implies that the young agent has only labour endowment and no capital endowment.

- The young agent in period t supplies his labour inelastically to the labour market in period t to earn a wage income w_t.

- Out of this wage income, the agent consumes a part and saves the rest - which becomes his capital stock **in the next period** and allows him to earn a rental income **in the next period**.

Representative Agent's Utility Function:

- Notice that since the agent would not be working in the next period, the savings and the consequent ownership of capital is the only source of income for him in the next period.

- Thus an agent is a worker in the first period of his life and becomes a capitalist (capital-owner) in the second period of his life.

- The young agent in period t optimally decides on his current consumption (c_t^1) and current savings (s_t) (or equivalently, his current consumption (c_t^1) and future consumption (c_{t+1}^2)) so as to maximise his lifetime utility:

$$U(c_t^1, c_{t+1}^2) \equiv u(c_t^1) + \beta u(c_{t+1}^2); \ 0 < \beta < 1, \tag{1}$$

where $u' > 0$; $u'' < 0$; $\lim_{c \to 0} u'(c) = \infty$; $\lim_{c \to \infty} u'(c) = 0$.

- β is the standard discount factor, which can be thought of as $\beta \equiv \dfrac{1}{1+\rho}$ where ρ represents the pure rate of time preference.

Representative Agent's Budget Constraints:

- The first period budget constraint of the agent:

$$c_t^1 + s_t = w_t.$$

- The second period budget constraint of the agent:

$$c_{t+1}^2 = (1 + r_{t+1}^e - \delta)s_t.$$

- Combining, we get the life-time budget constraint of the agent as:

$$c_t^1 + \frac{c_{t+1}^2}{(1 + r_{t+1}^e - \delta)} = w_t. \tag{2}$$

- The agent decides on his optimal consumption in the two periods by maximising (1) subject to (2).
- Since the agent is taking his savings decision in the first period, when second period's market interest rate is not yet known, he must optimize on the basis of some expected value of the r_{t+1}.
- As before, we shall assume that **agents have perfect foresight/rational expectations** such that $r_{t+1}^e = r_{t+1}$ for all t.

Representative Agent's Optimal Consumption & Savings:

- From the FONC of the optimization exercise:

$$\frac{u'(c_t^1)}{u'(c_{t+1}^2)} = \beta(1 + r_{t+1} - \delta). \tag{3}$$

- From the FONC and the life-time budget constraint, we can derive the optimal solutions as:

$$\begin{aligned} c_t^1 &= \psi(w_t, r_{t+1}); \\ c_{t+1}^2 &= \eta(w_t, r_{t+1}). \end{aligned}$$

- Corresponding optimal savings:

$$s_t = w_t - \psi(w_t, r_{t+1}) \equiv \phi(w_t, r_{t+1}).$$

- Before we proceed further, it is useful to note the signs of the partial derivatives s_w and s_r.

Representative Agent's Optimal Consumption & Savings (Contd.):

- Notice that

$$s_w \equiv \frac{\partial \phi(w_t, r_{t+1})}{\partial w_t} = 1 - \frac{\partial \psi(w_t, r_{t+1})}{\partial w_t} = 1 - \frac{\partial c_t^1}{\partial w_t}.$$

- From the life-time budget constraint of the agent:

$$\frac{\partial c_t^1}{\partial w_t} + \frac{1}{(1 + r_{t+1} - \delta)} \frac{\partial c_{t+1}^2}{\partial w_t} = 1.$$

- Under the assumption that both c_t^1 and $c_{t\_1}^2$ are normal goods, a unit increase in the wage rate w_t **ceteris paribus** must increase both c_t^1 and c_{t+1}^2. Thus $\frac{\partial c_t^1}{\partial w_t}, \frac{\partial c_{t+1}^2}{\partial w_t} > 0$.

- This in turn implies that

$$0 < s_w < 1.$$

Representative Agent's Optimal Consumption & Savings (Contd.):

- The sign of s_r however is ambiguous.
- Notice that

$$s_r \equiv \frac{\partial \phi(w_t, r_{t+1})}{\partial r_{t+1}} = -\frac{\partial \psi(w_t, r_{t+1})}{\partial r_{t+1}} = -\frac{\partial c_t^1}{\partial r_{t+1}}.$$

- So the sign of s_r depends on how first period consumption responds to a unit change in r_{t+1}.
- Recall however that $\dfrac{1}{1 + r_{t+1} - \delta}$ is the relative price of c_{t+1}^2 in terms of c_t^1.
- Thus a unit increase in r_{t+1} *ceteris paribus* **reduces** the relative price of future consumption in terms of current consumption.

Representative Agent's Optimal Consumption & Savings (Contd.):

- Any such price change will be associated with two effects:
  - a substitution effect ($\Rightarrow$ consumption should move in favour of the relatively cheaper good);
  - an income effect ($\Rightarrow$ the budget set of the consumer expands, which increases consumption of both goods)

- Thus due to an increase in r_{t+1} **ceteris paribus**
  - c_t^1 should decrease due to the substitution effect, while
  - c_t^1 should increase due to the income effect of a price change.

- The sign of $\dfrac{\partial c_t^1}{\partial r_{t+1}}$ depends on which effect dominates.

- This in turn implies that

$$s_r \quad \gtreqless \quad 0$$

according as, substitution effect $\gtreqless$ income effect.

Aggregate Consumption & Savings:

- Let us now turn our attention to the aggregate economy.
- Recall that in every period the total output is distributed as wage income and capital income:

$$Y_t = w_t N_t + r_t K_t. \tag{4}$$

- The entire wage income goes to the current young generation. Each of them saves a part of the wage income (s_t) and consume the rest. Thus,

$$w_t N_t = c_t^1 N_t + s_t N_t.$$

- On the other hand, the entire interest income goes to the current old generation. Each of them consume not only the interest earnings but the left over capital stock as well. Thus,

$$c_t^2 N_{t-1} = r_t K_t + (1 - \delta) K_t.$$

Aggregate Consumption & Savings (Contd.):

- Thus aggregate consumption in this economy at time t :

$$
\begin{aligned}
C_t &= c_t^1 N_t + c_t^2 N_{t-1} \\
&= [w_t N_t - s_t N_t] + [r_t K_t + (1-\delta)K_t] \\
&= w_t N_t + r_t K_t - \underbrace{[s_t N_t - (1-\delta)K_t]}_{S_t}
\end{aligned}
\tag{5}
$$

- Notice that aggregate savings S_t has two components:
  1. The positive savings by the young ($s_t N_t$);
  2. The negative savings by the old ($-(1-\delta)K_t$).

- As before, from the demand-supply equality for the aggregate economy:

$$
\begin{aligned}
C_t + I_t &= Y_t, \text{ where } I_t \equiv K_{t+1} - (1-\delta)K_t \\
&\Rightarrow K_{t+1} = s_t N_t.
\end{aligned}
$$

Dynamics of Capital-Labour Ratio:

- Since we know that labour force in this economy is growing at the rate n, i.e., $N_{t+1} = (1+n)N_t$, we can derive the dynamic of capital-labour ratio (k_t) as:

$$k_{t+1} = \frac{s_t}{(1+n)} = \frac{\phi(w_t, r_{t+1})}{(1+n)}. \qquad (6)$$

- Again, from the production side of the story, we already know that

$$
\begin{aligned}
w_t &= f(k_t) - k_t f'(k_t); \\
r_{t+1} &= f'(k_{t+1}).
\end{aligned}
$$

- Thus we can write (4) as:

$$k_{t+1} = \frac{\phi(w_t(k_t), r_{t+1}(k_{t+1}))}{(1+n)} = \Phi(k_t, k_{t+1}). \qquad (7)$$

- Equation (7) is the basic dynamic equation of the OLG model, which implicitly defines k_{t+1} as a function of k_t. Given k_0, we should be able to trace the evolution of the capital-labour ratio over time.
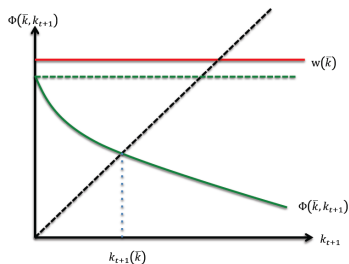
Existence of a Unique Perfect Foresight path:

- Let us look at dynamic equation (7). Notice that it is an 'implicit' difference equation with k_{t+1} entering on both sides.
- In fact this implicit nature of the function arises precisely due to assumption of perfect foresight. (Verify that with static expectation the difference equation is explicit and well-defined).
- The implicit function on the RHS poses a problem: for every k_t do we necceassrily get a **unique** k_{t+1} that satisfy the dynamic equation (7)?
- In other words, for any given initial value of k_0, **does a unique perfect foresight path exist**?
- Notice that uniqueness is important because otherwise the future trajectory of the economy will become indeterminate.

Existence of a Unique Perfect Foresight Path (Contd.):

- Notice that for any given value of k_t, say $\bar{k}$, from (7) we shall have a unique solution for k_{t+1} if and only if the curve representing $\Phi(\bar{k}, k_{t+1})$ has a unique point of intersection with the 45° line in the positive quadrant.

- A *sufficient* condition for this to happen is $\Phi(\bar{k}, k_{t+1})$ is either a flat line or is downward sloping with respect to k_{t+1}, i.e.,

$$\frac{\partial \Phi(\bar{k}, k_{t+1})}{\partial k_{t+1}} \leqq 0 \quad \text{for all } \bar{k} \in (0, \infty).$$

Existence of a Unique Perfect Foresight Path (Contd.):

- Notice that

$$
\begin{aligned}
\frac{\partial \Phi(\bar{k}, k_{t+1})}{\partial k_{t+1}} &= \frac{1}{(1+n)} \frac{\partial \phi(w_t, r_{t+1})}{\partial k_{t+1}} \\
&= \frac{1}{(1+n)} \frac{\partial \phi(w_t, r_{t+1})}{\partial r_{t+1}} \frac{dr_{t+1}}{dk_{t+1}} \\
&= \frac{1}{(1+n)} \, s_r \, f''(k_{t+1}).
\end{aligned}
$$

- Thus a *sufficient* condition for the existence of a unique perfect foresight path is: $s_r \geqq 0$.
- Henceforth we shall assume that this condition is satisfied (i.e., the utility function is such that the substitution effect of price change dominates the income effect).

Dynamics of Capital-Labour Ratio (Contd.):

- Having established that the difference equation given by (7) is well-defined, let us now characterise the evolution of k_t over time.
- Since $\Phi(k_t, k_{t+1})$ is a nonlinear function of k_t and k_{t+1}, we shall have to use the phase diagram technique to qualitatively characterise the dynamics.
- In drawing the phase diagram, first note that the slope of the phase line can be determined by total diffentiating (7):

$$(1 + n)dk_{t+1} = s_w \frac{dw_t}{dk_t} dk_t + s_r \frac{dr_{t+1}}{dk_{t+1}} dk_{t+1}$$

$$\text{i.e., } \frac{dk_{t+1}}{dk_t} = \frac{s_w \left[-k_t f''(k_t)\right]}{(1 + n) - s_r f''(k_{t+1})}.$$

- Under the assumption that $s_r \gtreqqless 0$, the slope of the phase line is necessarily positive.
- But even when the slope is positive, the curvature is not necessarily concave - since it would involve the third derivative of the utility function and the $f(k)$ function - whose signs are not known.

Stability & Dynamic Efficiency in the OLG Model:

- In other words, the nice result of the Solow model of a unique and globally stable steady state is no longer gauranteed - **despite the production function satifying all the standard neoclassical properties** - **including diminishing returns and the Inada conditions!**
- What about dynamic efficiency?
- Even that is not gauranteed any more!
- Below we provide an example - with specific functional forms - to show that dynamic efficiency is not necessarily gauranteed under the OLG model - despite optimizing savings behaviour by the agents.

Golden Rule & Dynamic Efficiency in the OLG Model:

- Before we turn to the specific functional forms, let us first define the golden rule in the context of the OLG model.
- As before let us define the 'golden rule' as that particular steady state which maximises the steady state level of per capita (average) consumption.
- Notice however that now there are two sets of people at any point of time t - current young (N_t) and current old (N_{t-1}).
- Hence per capita (average) consumption at any point of time t would be defined as:

$$c_t = \frac{c_t^1 N_t + c_t^2 N_{t-1}}{L_t} = \frac{c_t^1 N_t + c_t^2 N_{t-1}}{N_t + N_{t-1}} = \frac{(1+n)c_t^1 + c_t^2}{1 + (1+n)}.$$

- In other words, the per capita consumption in period t is the weighted average of the consumption of the current young and that of the current old.

Characterization of the Steady State in OLG Model:

- So what would be the steady state value of per capita consumption?
- Recall that the basic dynamic equation is the OLG model is given by:

$$k_{t+1} = \frac{s_t}{(1+n)} = \frac{\phi(w(k_t), r(k_{t+1}))}{(1+n)}$$

- Accordingly, the steady state(s) of the OLG model is defined as:

$$
\begin{aligned}
k^* &= \frac{s^*}{(1+n)} = \frac{\phi(w(k^*), r(k^*))}{(1+n)} \\
&\Rightarrow \quad s^* = (1+n)k^*
\end{aligned}
\tag{8}
$$

- On the other hand, from the optimal solutions of c^1 and c^2, we know that at steady state:

$$
\begin{aligned}
(c^1)^* &= \psi(w(k^*), r(k^*)) = w(k^*) - s^*; \tag{9} \\
(c^2)^* &= \eta(w(k^*), r(k^*)) = [1 + r(k^*) - \delta] s^*. \tag{10}
\end{aligned}
$$

Steady State Per Capita Consumption in the OLG Model:

- Hence from (1), (2) & (3) steady state per capita consumption:

$$
\begin{aligned}
c^* &= \frac{(1+n)\left(c^1\right)^* + \left(c^2\right)^*}{1+(1+n)} \\
&= \frac{(1+n)\left[w(k^*) - s^*\right] + \left[(1+r(k^*)-\delta)\right]s^*}{1+(1+n)} \\
&= \frac{(1+n)w(k^*) + \left[r(k^*)-\delta-n\right]s^*}{1+(1+n)} \\
&= \frac{(1+n)\left[\{w(k^*) + r(k^*)k^*\} - (n+\delta)k^*\right]}{1+(1+n)} \\
&= \frac{1+n}{1+(1+n)}\left[f(k^*) - (n+\delta)k^*\right].
\end{aligned}
$$

- Maximizing c^* with respect to k^*, we would still get the golden rule condition as:

$$
k_g : f'(k^*) = (n+\delta)
$$

Alternative Definition of Golden Rule:

- An alternative definition of the 'golden rule' in the context of the OLG model can be as follows: it is that particular steady state which maximises the steady state level of **utility** of any agent.
- When the economy is at a steady state both w and r become constants. Thus for any agent belonging to any generation ($t-1$ or t or $t+1$), the life-time consumption profile look exactly the same:

$$
\begin{aligned}
c_{t-1}^1 &= c_t^1 = c_{t+1}^1 = \psi(w(k^*), r(k^*)) \equiv \left(c^1\right)^*; \\
c_t^2 &= c_{t+1}^2 = c_{t+2}^2 = \eta(w(k^*), r(k^*)) \equiv \left(c^2\right)^*.
\end{aligned}
$$

- Hence steady state utility of any generation is given by:

$$
U(\left(c^1\right)^*, \left(c^2\right)^*) \equiv u(\left(c^1\right)^*) + \beta u(\left(c^2\right)^*).
$$

- It is easy to check that the k^* that maximises $U(\left(c^1\right)^*, \left(c^2\right)^*)$ is still given by:

$$
k_g : f'(k^*) = (n + \delta).
$$

- Thus the two definitions of the golden rule are equivalent. (**Verify.**)

OLG Model with Specific Functional Forms:

- Let us now assume specific functional forms.
- Let

$$U(c_t^1, c_{t+1}^2) \equiv \log c_t^1 + \beta \log c_{t+1}^2;$$

$$f(k_t) = (k_t)^\alpha; \ 0 < \alpha < 1.$$

- Also let the rate of depreciation be 100%, i.e., $\delta = 1$.
- Thus the life-time budget constraint of the representative agent of generation t is given by:

$$c_t^1 + \frac{c_{t+1}^2}{r_{t+1}} = w_t.$$

- The corresponding FONC:

$$\frac{c_{t+1}^2}{c_t^1} = \beta r_{t+1}. \tag{11}$$

OLG Model: Specific Functional Forms (Contd.)

- From the FONC and the life-time budget constraint, we can derive the optimal solutions as:

$$
\begin{aligned}
c_t^1 &= \frac{1}{1+\beta} w_t; \\
s_t &= \frac{\beta}{1+\beta} w_t; \\
c_{t+1}^2 &= \beta r_{t+1} \left[\frac{1}{1+\beta} w_t \right].
\end{aligned}
$$

- Corresponding dynamic equation:

$$
k_{t+1} = \frac{s_t}{(1+n)} = \left(\frac{1}{1+n} \right) \left[\frac{\beta}{1+\beta} w_t \right].
$$

- Finally, given the production function,

$$
w_t = (1-\alpha) \left(k_t \right)^\alpha
$$

OLG Model: Dynamics with Specific Functional Forms

- Thus the dynamics of this specific case is simple:

$$k_{t+1} = \left(\frac{1}{1+n}\right)\left(\frac{\beta}{1+\beta}\right)(1-\alpha)(k_t)^{\alpha}.$$

- It is easy to see that the above phase line will generate a unique non-trivial steady state which is globally stable.

- The corresponding steady state solution is defined as:

$$k^* = \left(\frac{1}{1+n}\right)\left(\frac{\beta}{1+\beta}\right)(1-\alpha)(k^*)^{\alpha}$$

$$\text{i.e., } k^* = \left[\left(\frac{1}{1+n}\right)\left(\frac{\beta}{1+\beta}\right)(1-\alpha)\right]^{\frac{1}{1-\alpha}}.$$

- Is this steady state dynamically efficient? Not necessarily!

OLG Model: Dynamic Inefficiency

- Notice that given the specific functional form, the 'golden rule' value of the capital-labour ratio can be derived as:

$$
\begin{aligned}
k_g &: \quad f'(k^*) = (n + \delta) \\
\text{i.e., } k_g &: \quad \alpha(k^*)^{\alpha-1} = 1 + n \\
\text{i.e., } k_g &= \quad \left[\frac{\alpha}{1+n} \right]^{\frac{1}{1-\alpha}}.
\end{aligned}
$$

- It is easy to verify that the steady state under this specific example will be dynamically inefficient whenever

$$
\frac{\beta}{1+\beta} > \frac{\alpha}{1-\alpha}.
$$

- Example: $\beta = \dfrac{1}{2}$; $\alpha = \dfrac{1}{5}$.

Reason for Dynamic Inefficiency in the OLG Model:

- Thus we see that dynamic inefficiency may arise in the OLG model - despite optimizing savings behaviour by households.
- This apparently paradoxical result stems from the fact that agents are 'selfish' in the OLG model; they do not care for their children's utility/consumption.
- Thus when they optimise they equate the MRS with (the discounted value of) the actual future return $(r_{t+1} + 1 - \delta)$:

$$\frac{u'(c_t^1)}{u'(c_{t+1}^2)} = \beta(r_{t+1} + 1 - \delta).$$

- Now notice that

$$\frac{u'(c_t)}{u'(c_{t+1})} \gtreqless 1 \Rightarrow u'(c_t) \gtreqless u'(c_{t+1}) \Rightarrow c_t \lesseqgtr c_{t+1}.$$

- This implies that in the OLG framework, $c_t \lesseqgtr c_{t+1}$ i.e, consumption would rise, fall or remain unchanged over time if the corresponding (future) return, measured by $(r_{t+1} + 1 - \delta)$ is greater, equal to or less than the subjective cost $\frac{1}{\beta} \equiv (1 + \rho)$, ie. according as

$$r_{t+1} - \delta \gtreqless \rho.$$

- Recall that in the R-C-K model the corresponding equation was given by:
$$\frac{dc}{dt} = \frac{c_t}{\sigma(c_t)} \left[r_t - \delta - n - \rho \right].$$

- Thus in the R-C-K framework, $\frac{dc}{dt} \gtreqless 0$ i.e, consumption would rise, fall or remain unchanged over time if the corresponding **population-adjusted** (future) return, measured by $(r_t - \delta - n)$ is greater, equal to or less than the subjective cost measured by ρ.

Reason for Dynamic Inefficiency in OLG Model (Contd.)

- Notice that both in the OLG model and the R-C-K model, households will stop saving whenever their percieved return is less than the subjective cost ρ.
- But due to presence of intergenerational altrauism, in the R-C-K model the percieved return is adjusted for popultion growth and is given by $r - \delta - n$, whereas in the OLG model this return is just $r - \delta$
- This implies that in the R-C-K model households will *necessarily* stop saving when the (net) marginal product has fallen below the population growth rate ($r_t - \delta - n < 0$).
- But there is no reason why in the OLG model, households would stop saving in this scenario, because even though the net gross marginal product ($r_{t+1} - \delta$) has fallen below n - it might still greater than the subjective cost ρ.
- Thus there is a tendency to oversave (compared to the R-C-K model), which persists even when the economy has moved into a dynamically inefficient region (i.e., $r_{t+1} < \delta + n$).

Dynamic Inefficiency & Scope for Government Intervention in the OLG Model:

- Since under the OLG framework, the steady state of the decentralized market economy may be dynamically inefficient (despite rational expectations on part of the agents), this again justifies a role of government in improving efficiency.

- Thus the conclusions of the OLG model are diametrically opposite of that of the R-C-K model - even though both are based on strictly neoclassical production function and optimizing agents.

- The difference arises primarily due to the absence of parental altruism in the OLG framework.

- It can be shown that if we introduce parental altruism in the OLG model (by incorporating a bequest term that each parent leaves to his child at the end of his life time), then the OLG framework will be very similar to the R-C-K framework.

OLG Model: Summary

- Even though the production function is Neoclassical in the OLG model, the strong stability result of Solow (as well as R-C-K model) does not hold:
  - a steady state may not exists;
  - even if it exists it may not be unique;
  - even if it is unique, it may not be stable.
- So the growth conclusions of the Solow model do not hold either.
- Moreover the dynamic inefficiency problem may reappear here- even though each agent is optimally determining his savings behaviour.
- Thus in this model there is scope for government intervention in terms of improving efficiency.
- Since the OLG model does not obey the growth properties of the Solow/R-C-K model, we do not identify this structure with the "Neoclassical Growth Model" (even though the production structure here is identical to Solow/R-C-K).

References:

- Reference for the OLG Model:
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