COURSE 801: INDUSTRIAL ECONOMICS
MIDTERM EXAM, 10.9.18

Time: 70 minutes **30 Marks**

Answer Question 1 and <u>one</u> other question. Keep your answers short and precise, taking care to bring out the relevance of the various assumptions of each model at the appropriate stages of your derivation.

1. (COMPULSORY)

a) Suppose that there are two identical firms (1 and 2) which produce a homogenous product with zero marginal costs. They compete in prices and sell to a market with inverse demand given by $P = 2 - Q$. Suppose that prices are constrained to take only integer values. Using standard game-theoretic reasoning, determine whether or not each possible price pair (p_1, p_2) can be regarded as a Bertrand-Nash equilibrium of a one-period game. (You can ignore prices ≥ 2. Assume that demand is equally split if prices are equal.)

b) Now suppose that prices are continuous, so that we get the standard Bertand model. Firms 1 and 2 compete in two identical markets A and B. However, there is another firm (firm 3) which is present in market B only. All firms have the same costs in each market, so the Nash equilibrium in each market is the Bertrand paradox. The firms compete over an infinite horizon, discounting future profits at the rate δ. They can try to form a cartel supported by grim-trigger punishment in the form of Nash reversion to punish defection without any lag. Set up and explain the relevant incentive-compatibility conditions and find the minimum discount factor that would support collusion (i) in each market separately; and (ii) in both markets if the firms use cross-market retaliation to punish defection.

(5, 10)

2. Use the following information to answer parts (a) and (b) of this question. Firms 1, 2 and 3 produce differentiated products with zero marginal costs. Inverse demand for the i^{th} firm is given by

$$p_i = a - q_i - d\sum_{j \neq i} q_j, \qquad \text{where } a > 0 \text{ and } 0 < d < 1.$$

The firms compete in quantities for a single period. In this model:

a) Determine the Cournot-Nash equilibrium quantities that each firm will produce if they compete against each other as a 3-firm oligopoly.

b) Show that a cartel consisting of all three firms will be internally unstable, i.e. any representative firm will prefer to stay out of the cartel, assuming that the remaining two firms form a cartel and behave like a single firm. (Assume that cartels always share their maximized profits equally.) Explain the intuition behind this result in terms of an externality.

(5, 10)

3. A good is demanded by a unit mass of identical consumers who are uniformly distributed along a line segment of unit length, representing a product characteristic. Each consumer has inelastic unit demand with willingness to pay of v, which is high enough to ensure that the market is covered in equilibrium. The good is produced by two identical firms 1 and 2 with marginal cost c, located at l_1 and l_2 on the line segment, where $0 < l_1 < l_2 < 1$. To buy the good, a consumer must incur transport costs (disutility) that is a *quadratic* function td^2 of the distance d to a firm's location. The firms simultaneously choose prices, given their locations.
   a) In this setting, determine (i) whether this is a case of demand substitutability or complementarity, and (ii) whether prices are strategic substitutes or strategic complements.
   b) Determine the Bertrand-Nash equilibrium prices and profits, and show that the Bertrand Paradox emerges as a special case with no product differentiation.
   c) If the firms can simultaneously choose their locations before competing in prices, prove that they will maximally differentiate their products. Explain this result in terms of the 'competition effect' and the 'market size effect'.

(5, 5, 5)