Biological foundation of economic choice

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Lecture 3

NeuroEconomics as I see it

Main message

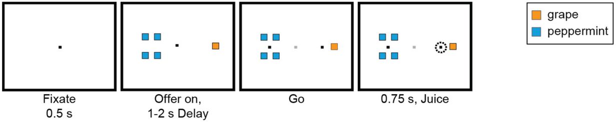
A biological foundation of economics is possible **now**.

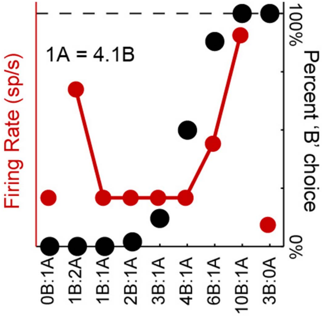
Program

1. **Replace "as if" models with mechanistic models**; we care about the realism of the assumption, and we do not take prediction out of sample as the only criterion to evaluate a theory
2. **Understand genetic determinants of pathways of personality and choice**
3. **Expand the domain of human personality we consider relevant** Francis Hutcheson *"There is no part of Philosophy of more importance, than a just knowledge of Human Nature, and its various Powers and Dispositions"*; David Hume *"There is no question of importance, whose decision is not684 compriz'd in the science of man"*

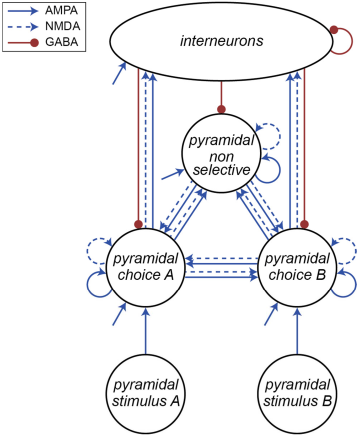
Let's consider an example of a mechanistic model

Padoa-Schioppa Assad, 2006: subjects, task and data

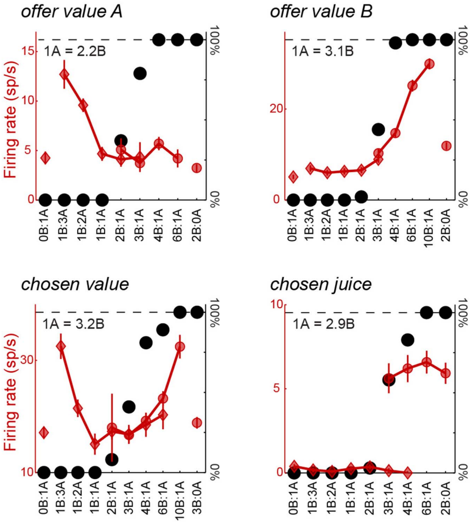




A general neuro-computational model

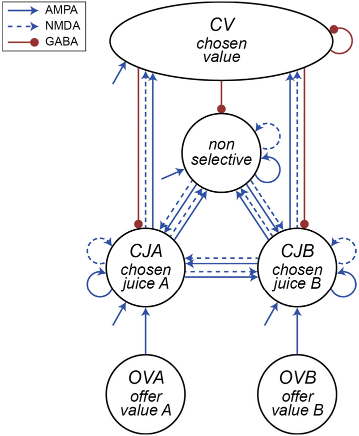


Three types of neurons



Economic Choice Model

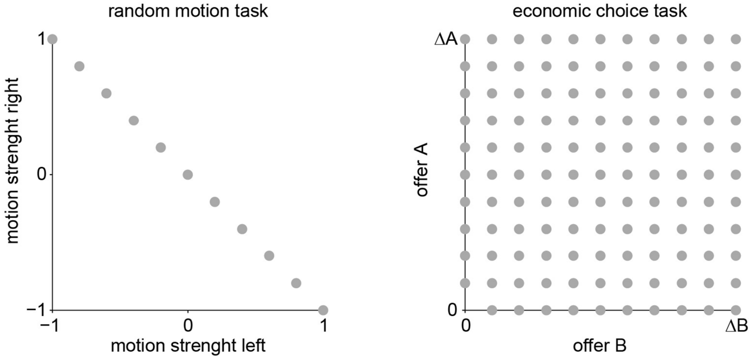
Wong & Wang 2006, Rustichini & Padoa-Schioppa 2015, Rustichini et al., 2018



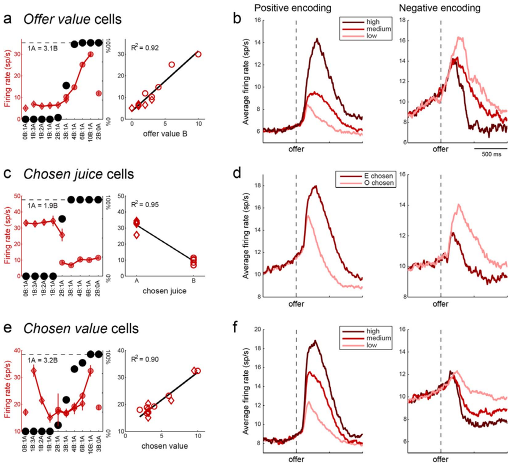
Gating variables model, 2D

1. S^i is the fraction of gating variables (NMDA receptors) open for good i ;
2. $$\frac{dS^i}{dt}(t) = -\frac{S^i(t)}{\tau} + (1 - S^i(t))\gamma\phi(X^i),$$
   - for $i = A, B$, where for $i = A, B, j \neq i$,
     $$X^i \equiv J^{ii} S^i(t) - J^{ij} S^j(t) + I^i(t)$$
   - $J^{ii}, J^{ij} > 0$
   - I^i is the input for the option i.

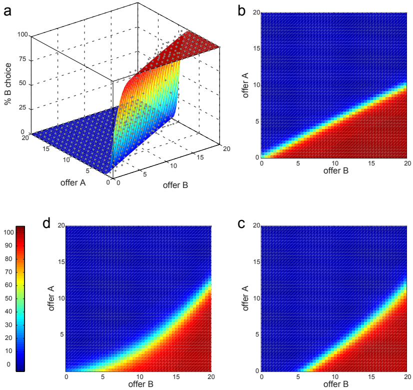
Economic choices are harder



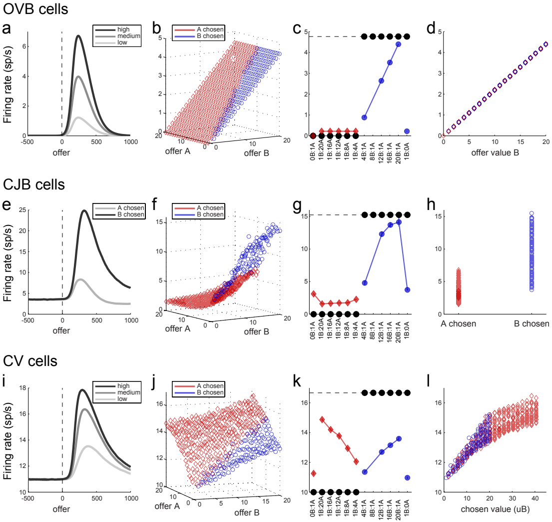
Choice and firing patterns in experimental data



Probability of choice



Match of real data and model



Now we want theorems like these:

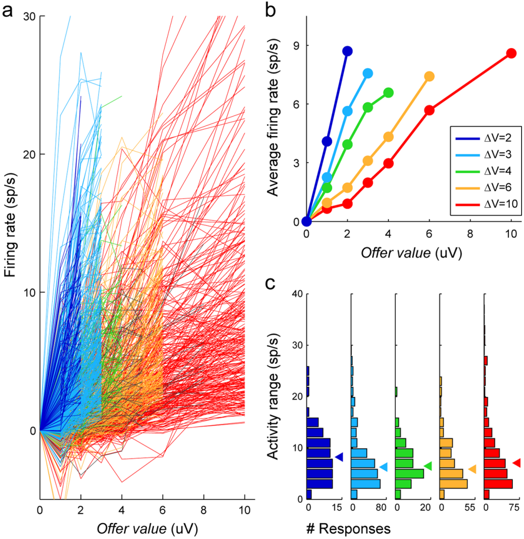
Cardinal Utility

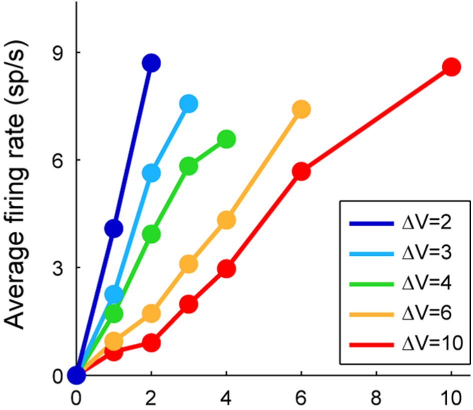
Theorem we have a cardinal utility **because** neurons communicate information in an additive way

Adaptive Coding

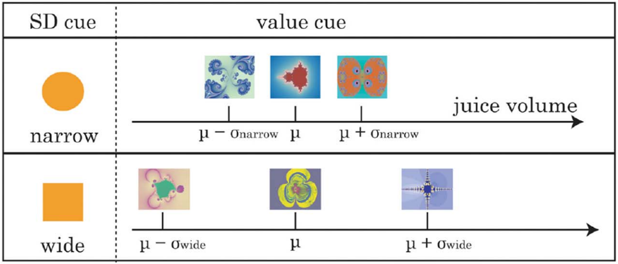
Theorem we have adaptive coding **because** the spike process is Poisson

Adaptive Coding (Padoa-Schioppa, 2009)




Average firing rate (sp/s)
9
6
3
0
0
2
4
6
8
10
ΔV=2
ΔV=3
ΔV=4
ΔV=6
ΔV=10

Kobayashi, de Carvalho, Schultz, 2010



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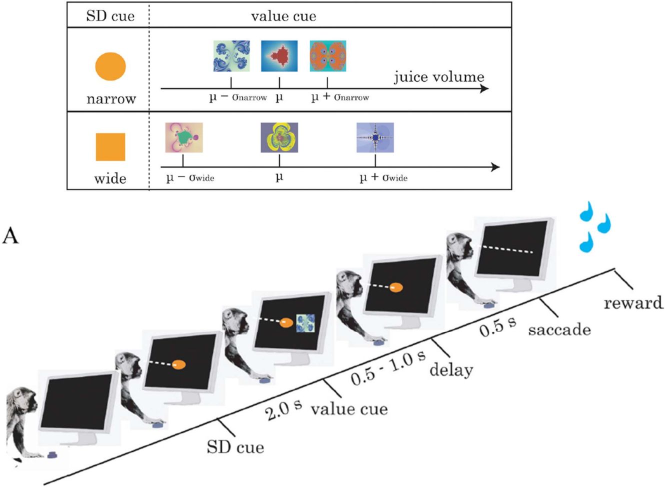

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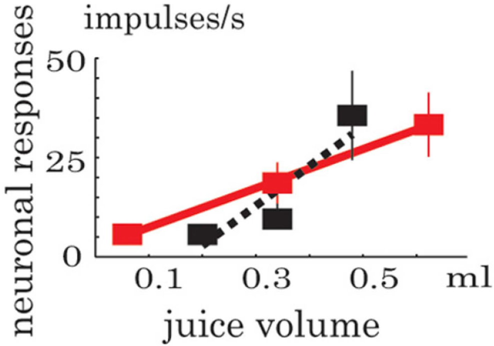


Why Adaptive coding: how not to find an answer (Wong & Wang 2006)

The mean-field approach does not include time-varying noise that plays a critical role in the spiking neural network. To amend this, we added a noise term I_{noise} implemented as a white noise filtered by a short (AMPA synaptic) time constant. This is thus described by an Ornstein–Uhlenbeck process (Uhlenbeck and Ornstein, 1930) (for example, see Destexhe et al., 2001):

$$\tau_{\text{AMPA}} \frac{dI_{\text{noise}}(t)}{dt} = -I_{\text{noise}}(t) + \eta(t)\sqrt{\tau_{\text{AMPA}}\sigma_{\text{noise}}^2},$$

where σ_{noise}^2 is the variance of the noise, and η is a Gaussian white noise with zero mean and unit variance. Unless specified, σ_{noise} is fixed at 0.007 nA.

- Wang's model ignores the dependence on mean and SD on firing rate, assuming an input equal to the sum of a constant plus a constant coefficients *OU* process;
- Adaptive coding in this way is either useless or impossible
- We want to explain it from first principles

The rule of the game

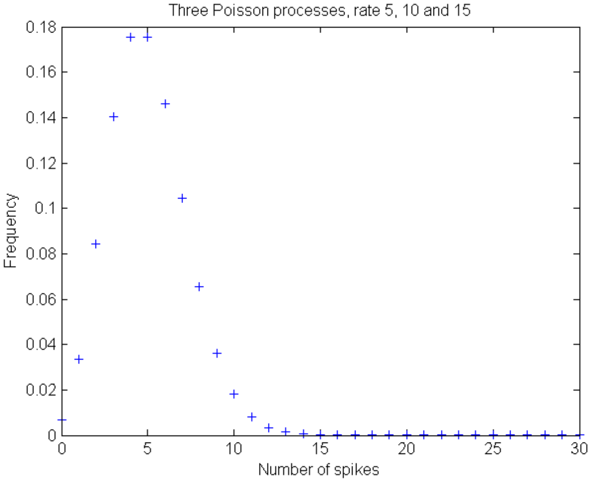
- Voltage in a neuron evolves according to

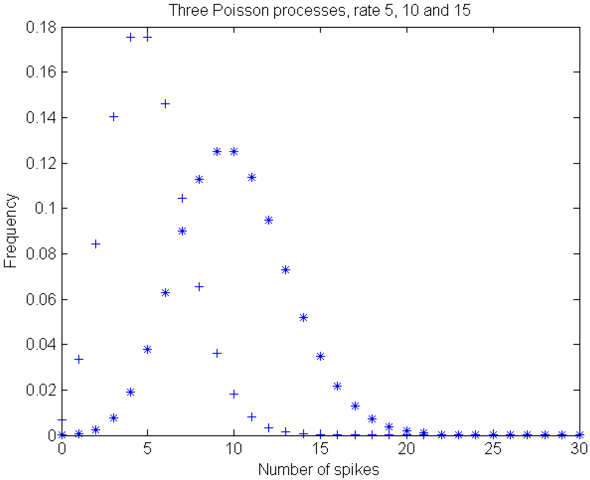
$$\begin{aligned} V(t+dt) - V(t) &= -g(V(t) - V_L)dt \\ &+ J_E(N_E(t+dt) - N_E(t)) - J_I(N_I(t+dt) - N_I(t)) \end{aligned}$$

  J's are current inputs, N's are Poisson processes
- When $V(t)$ reaches a threshold value V_{th}, the neuron fires,
- The voltage is reset to a V_r value and the process starts again
- We want to explain adaptive coding with just these elements:

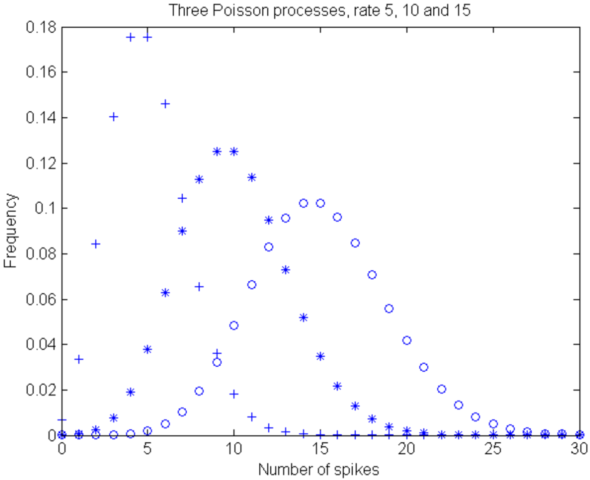
  Theorem *We have adaptive coding* **because** *the spike process is Poisson*

Poisson process

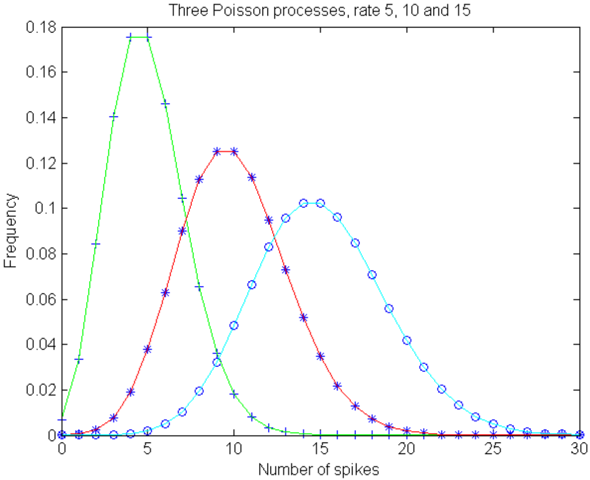



Three Poisson processes, rate 5, 10 and 15
Frequency
Number of spikes
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30

Poisson process



It all follows from the Poisson property Mean = Variance



What does this property do for you? A simple example

1. Two offers in quantities (x, y), with known joint symmetric distribution $\pi \in \Delta(X \times Y)$
2. You do not observe the (x, y), but a signal on it
3. You can choose one of the two options (action set $\{1, 2\}$);
4. Two signals $\mathbf{X} \sim N(sx, sx)$ and $\mathbf{Y} \sim N(sy, sy)$; you only observe the difference between the two,
$$D \equiv \mathbf{X} - \mathbf{Y}$$
5. **You want to get the largest expected payoff**; to do this you can pick any $s \in S$ (say s smaller than a given maximum), and after the observation of D you can choose 1 or 2

A very simple example

1. The optimal choice of action conditional on signal (by symmetry) is: *choose* 1 *if and only if the signal* D *is positive*
2. Let $Q_{(x,y)}(\cdot|s)$ the probability of D given the pair (x, y) and slope s; note that for a given pair (x, y) and slope s

$$D \sim N(s(x-y), s(x+y))$$

3. By the point 1. above:

$$\text{probability of choosing 1 at } ((x,y),s) = Q_{(x,y)}(R^+|s)$$

4. The largest expected payoff is

$$\max_{s \in S} \int_{X \times Y} \left(Q_{(x,y)}(R^+|s)x + Q_{(x,y)}(R^-|s)y \right) d\pi =$$

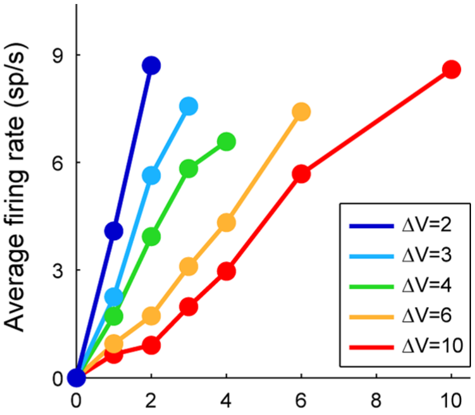
$$\int_{X \times Y} y d\pi + \max_{s \in S} \int_{X \times Y} Q_{(x,y)}(R^+|s)(x-y) d\pi$$

so we focus on $Q_{(x,y)}(R^+|s)$

A very simple example

1. $$\begin{aligned} Q_{(x,y)}(R^{+}|s) &= P\left(Z \geq -\frac{\text{mean}}{\text{standard deviation}}|Z \sim N(0,1)\right) \\ &= P\left(Z \geq -\frac{(x-y)\sqrt{s}}{\sqrt{x+y}}|Z \sim N(0,1)\right) \end{aligned}$$

   so increasing s makes the probability of choosing 1 when $x \geq y$ larger, and the probability of choosing 1 when $x \leq y$ smaller.
2. Hence the payoff increases with s and **the optimal policy is to choose s as large as possible.**
3. BUT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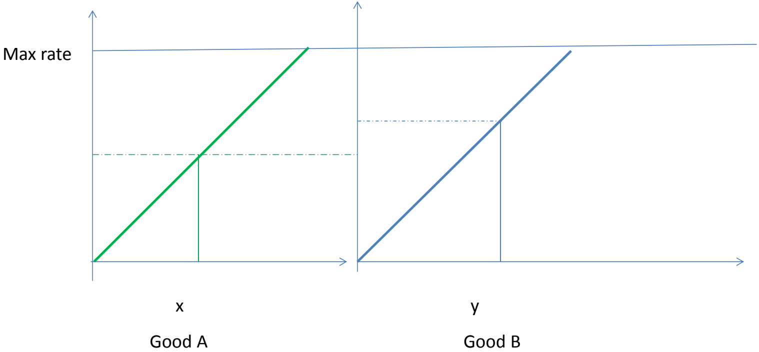
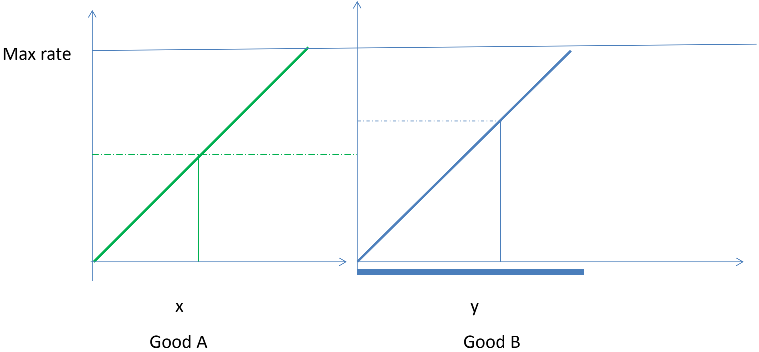
A potential problem



A potential problem



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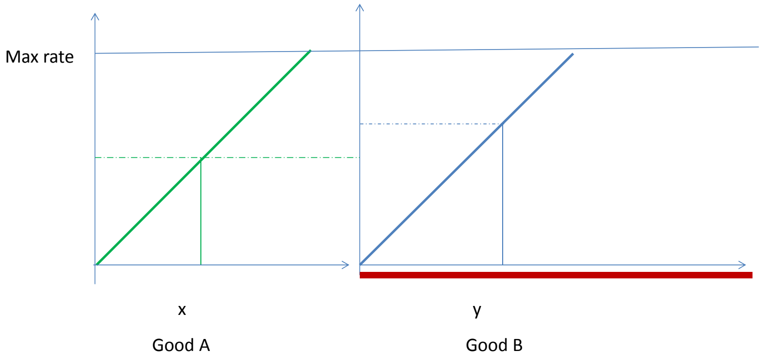
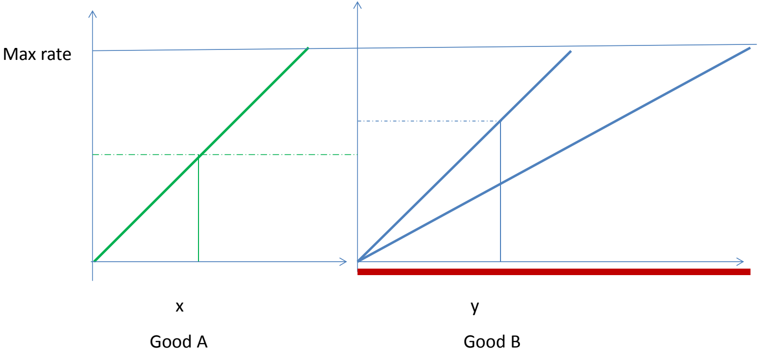
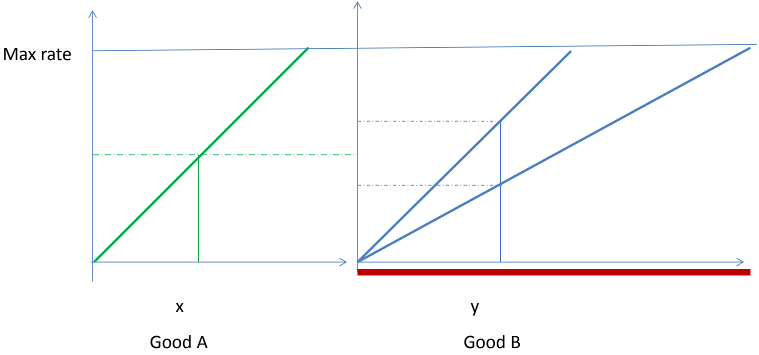


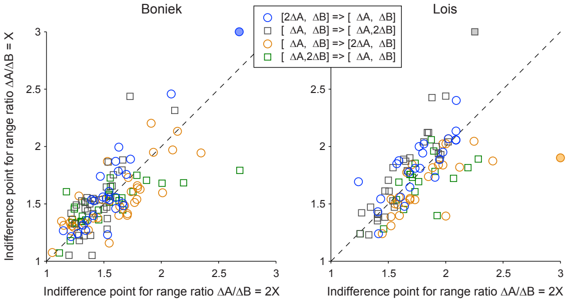
A potential problem



A potential problem: HUGE environment bias (?)



But is the bias there?



The real neural problem

1. The input to the downstream neuron are the product of spikes per unit of time, times inputs (J_A and J_B) that depend on the session not on the trial offer
2. Assume the firing rate is linear in the quantity offered, *eg* for good A

$$\text{firing rate } = s_A x$$

3. (Britten et al., VN, 1995): *A relationship between behavioral choice and the visual response of neurons in macaque MT*.
4. Neurons do not fire independently; correlation coefficient $\rho > 0$; higher ρ, higher variance of the signal.

$$\max_{(s_A, J_A, s_B, J_B)} \int_{X \times Y} P\left(Z \geq -\frac{\text{mean}}{\text{standard deviation}} | Z \sim N(0,1)\right)(x-y)d\pi(x,y)$$

where

$$\begin{aligned} \frac{\text{mean}}{\text{standard deviation}} &= \frac{s_A x J_A - s_B y J_B}{(\rho(s_A x J_A^2 + s_B y J_B^2))^{1/2}} \\ &= \frac{s_A x - s_B y R}{(\rho(s_A x + s_B y R^2))^{1/2}} \end{aligned}$$

$$R \equiv \frac{J_B}{J_A}; s_A x \equiv \text{ firing rate for good A}, J_A \equiv \text{ input for good A}$$

subject to:

$$\forall g \in \{A, B\},\ 0 \leq s_g \leq \frac{\bar{s}}{M_g}, R \geq 0.$$

$$\pi \text{ uniform on } [0, M_A] \times [0, M_B]$$

Optimal solution

1. The slopes are at the maximum values,

$$s_g = \frac{\bar{s}}{M_g}$$

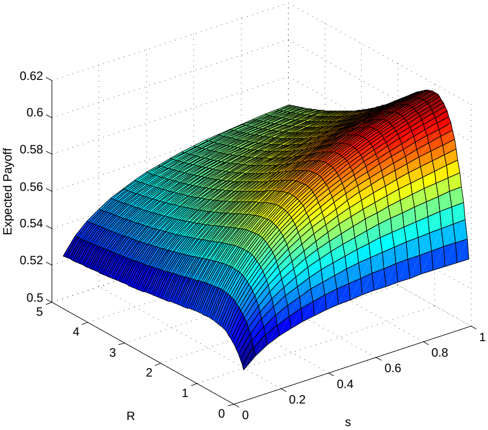
2. R adjusts to compensate and reduce the difference

$$s_A J_A - s_B J_B$$

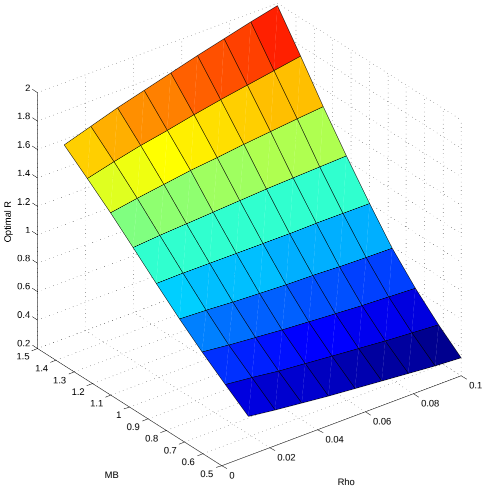
   to close to zero,

3. large values of ρ or large differences between M_A and M_B introduce a bias
4. The bias is optimal in extreme cases

Optimal firing rates and inputs: symmetric case, $s_A = s_B \equiv s$



Optimal ratio R, range of good B and neuronal correlation



Optimal ratio R versus range of good B