Delhi School of Economics Winter School

Epidemics, growth and economic behavior:
Traditional approaches and new covid-driven research

2. Epi-econ modelling: Basic models

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More Covid-19 special issues...

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Editorial

The economics of epidemics and contagious diseases: An introduction

0. Introduction

In December 2019, medical professionals hypothesized that clusters of a new respiratory disease were developing in the Chinese city of Wuhan. On January 3rd, 2020, the World Health Organization confirmed this hypothesis, as 44 cases were diagnosed as SARS-CoV-2 infections, the cause of the COVID-19 disease. Despite multiple measures to contain it, the virus spread beyond China's borders, and in January, the first case outside the country was confirmed in Thailand. By the end of the month, around 8000 cases and 170 deaths had been confirmed in different countries. By mid-July, the need for ICU beds in the Italian

quarter of them, while the same indicator was less than 2% for half of the households in the fifth quintile.[5] And across sectors of economic activity, the variability in the impact of the pandemic is similarly wide: In the US, four weeks after the first human-to-human contagion was reported on January 30th, household demand for dried beans had increased by 37%, and job openings for interpreters and translators had increased three-fold[6]; three weeks later, sales at restaurants had decreased by 47%, and 70% of the restaurants had laid off employees, while demand for childcare workers had contracted by 36%.[7] In their most *optimistic* scenario, Škare et al. (2021) predict that over 160 million jobs will be lost worldwide in the tourism sector.

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INTRODUCTION

JOURNAL OF PUBLIC ECONOMIC THEORY WILEY

Introduction to the special issue on new insights into economic epidemiology: Theory and policy

While the entire planet seems to coexist with COVID without major problems at present, as almost all the preventive measures have been lifted, there is increasing concern among epidemiologists and expert scientists that the worse of COVID-19 might not yet be over. Uncertainty is still significant to the point that one cannot rule out the possibility that even the severe form of the pandemic experienced in 2020 will emerge again next Fall. The bulk of the uncertainty resides in the dangerousness of the mutating virus and the efficacy of the new releases of the RNA vaccines to fight these mutations. World Health Organization (WHO) and European Medicine Agency (EMA) are voicing these concerns regularly on their respective

Epi-econ: a not-so-old brand name

- **Epi-econ models refer to models merging epidemiological diffusion and economic models**. The preceding economic epidemiology literature does not model epidemic contagion, and typically reduce epidemics to pointwise or enduring mortality/morbidity shocks which effects may be in certain cases controled with some instruments (like health expenditures). See e.g Chakraborty and Das (2005) or Boucekkine et al. (2009).
- As controling epidemic diffusion becomes key with the Covid shock (lockdowns, masks, testing,...), almost all the economists become epidemiologists in Spring 2020. Models like the SIS, SIR, SEIRD,...etc, have now become routine.
- First epi-econ modelling is due to Geoffard and Philipson (1996) and Gersovitz and Hammer (2004) is often identified as such. Unconnected to development issues.
- **Another pioneer in this field is Goenka, Lin and co-authors (ET 2012, JME 2014, ET 2020, JME 2021,...) who developed the first epi-econ growth models**.

The simplest epidemiological models

- Continuous-time SIS epidemiology model:

$$dS/dt = bN - dS - \alpha SI/N + \gamma I$$
$$dI/dt = \alpha SI/N - (\gamma + d)I$$
$$dN/dt = (b - d)N$$

- Continuous-time SIR model

$$dS/dt = bN - (d + d_s)S - \alpha SI/N$$
$$dI/dt = \alpha SI/N - (\gamma)R - (d + d_i)I$$
$$dR/dt = \gamma R - (d + d_r)R$$
$$dN/dt = (b - d)N - d_s S - d_i I - d_r R$$

Gersovitz and Hammer, 2004

The Economic Journal, **114** (*January*), 1–27. © Royal Economic Society 2004. Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main Street, Malden, MA 02148, USA.

THE ECONOMICAL CONTROL OF INFECTIOUS DISEASES*

Mark Gersovitz and Jeffrey S. Hammer

The structure of representative agents and decentralisation of the social planner's problem provide a framework for the economics of infection and associated externalities. Optimal implementation of prevention and therapy depends on: (1) biology including whether infection is person to person or by vectors; (2) whether the infected progress to recovery and susceptibility, immunity, or death; (3) costs of interventions; (4) whether interventions target everyone, the uninfected, the infected, or contacts between the two; (5) individual behaviour leading to two types of externalities. By way of example, if people recover to be susceptible, government subsidies should equally favour prevention and therapy.

About Gersovitz and Hammer, 2004

- GH is the first paper which aims at producing a *general* epi-econ model with a careful analysis of the externalities involved and optimal public health policies in the face of epidemic outbreaks.
- GH consider a compartmental epidemiological model with Susceptibles, Infected/Infectious and Uninfectibles (recovered + immunized), and a government using optimally prevention and treatment to maximize an intertemporal social welfare function accounting for the cost of these control policies and the economic losses due to infections.
- Prevention and curative affect the infection and other parameter of the epidemic dynamics.
- A pure health economics (though intertemporal model) with no factor accumulation.
- Limited applicability to Covid: the paper is about prevention vs treatment from t=0. Does not cover all aspects of Covid-like epidemics (in particular asymptomatic).

An example from the recent Covid literature: Caulkins et al., 2021)

The state dynamics can then be written as

$$\dot{S}(t) = \nu N(t) - \beta(\gamma(t))\frac{S(t)I(t)}{N(t)} - \mu S(t) + \varphi R(t)$$

$$\dot{I}(t) = \beta(\gamma(t))\frac{S(t)I(t)}{N(t)} - (\alpha + \mu + \mu_I)I(t)$$

$$\dot{R}(t) = \alpha I(t) - \mu R(t) - \varphi R(t)$$

Outlines of this lecture

1. Introduction: From the start to the takeoff of the epi-econ modelling
2. Seminal epi-econ growth model(s)
3. The great variety of Covid-driven epi-econ modelling
4. An example from the first-generation of the epi-econ Covid models
5. Concluding remarks

The epidemiology model

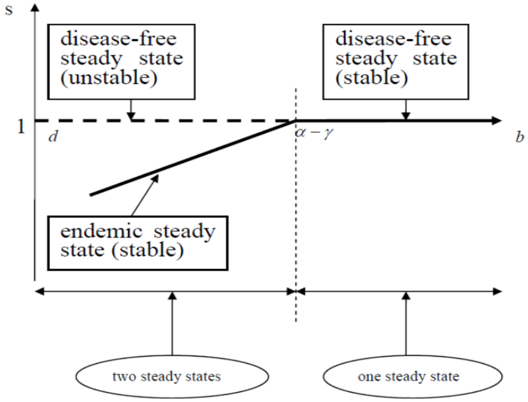
- Continuous-time SIS epidemiology model:

$$
\begin{aligned}
dS/dt &= bN - dS - \alpha SI/N + \gamma I \\
dI/dt &= \alpha SI/N - (\gamma + d)I \\
dN/dt &= (b - d)N
\end{aligned}
$$

- In terms of $s_t = S_t/N_t$,

$$\dot{s}_t = (1 - s_t)(b - \alpha s_t + \gamma)$$

Steady state and stability properties of SIS (Hethcote, 2009)



Epidemiological parameters

- Endogenize epidemiological parameters by making them depend on health capital, h.

Assumption (2)

- $\alpha(h_t)$ *is a* C^∞ *function with* $\alpha'(h_t) < 0$, $\alpha''(h_t) > 0$, $\lim_{h_t \to 0} \alpha'(h_t) < \infty$, $\lim_{h_t \to \infty} \alpha'(h_t) \to 0$, $\alpha(h_t) \to \overline{\alpha}$ *as* $h_t \to 0$ *and* $\alpha(h_t) \to \underline{\alpha}$ *as* $h_t \to +\infty$;
- $\gamma(h_t)$ *is a* C^∞ *function with* $\gamma'(h_t) > 0$, $\gamma''(h_t) < 0$, $\lim_{h_t \to 0} \gamma'(h_t) < \infty$, $\lim_{h_t \to \infty} \gamma'(h_t) \to 0$, $\gamma(h_t) \to \underline{\gamma}$ *as* $h_t \to 0$ *and* $\gamma(h_t) \to \overline{\gamma}$ *as* $h_t \to +\infty$.

Labor supply

- Infected people cannot work and labor force consists only of healthy people with labor supplied inelastically
- $L(t)$ inherits the dynamics of $S(t)$:

$$\dot{I}_t = (1 - I_t)(b + \gamma - \alpha I_t)$$

Production

Assumption (3)

The production function $f(k_t, l_t) : \Re^2_+ \to \Re_+$*:*

1. $f(\cdot,\cdot)$ *is* C^∞ *and homogenous of degree one;*
2. $f_1 > 0, f_{11} < 0,\ f_2 > 0, f_{22} < 0,\ f_{12} = f_{21} > 0$ *and* $f_{11}f_{22} - f_{12}f_{21} > 0$*;*
3. $\lim_{k_t \to 0^+} f_1 = \infty,\ \lim_{k_t \to \infty} f_1 = 0$ *and* $f(0, l_t) = f(k_t, 0) = 0$*.*

Assumption (4)

The production function $g(m_t) : \Re_+ \to \Re_+$ *is* C^∞ *with* $g' > 0,\ g'' < 0,$ $\lim_{m_t \to 0} g' < \infty$ *and* $g(0) = 0$*.*

Production

- Standard two sector growth model: physical goods and health are generated by different production functions.

$$\dot{k}_t = f(k_t, l_t) - c_t - m_t - \delta k_t - k_t(b-d)$$
$$\dot{h}_t = g(m_t) - \delta h_t - h_t(b-d)$$

Preferences

- Utility function depends only on current consumption, c_t, is additively separable, and is discounted at the rate $\theta > 0$.

Assumption (5)

The instantaneous utility function $u(c_t) : \Re_+ \rightarrow \Re_+$ *is* C^∞ *with* $u' > 0$, $u'' < 0$ *and* $\lim_{c_t \rightarrow 0^+} u' = \infty$.

The Social Planner's Problem

The optimization problem for social planner:

$$\max_{\{c_t, m_t\}} \int_0^\infty e^{-\theta t} u(c_t) dt \tag{1}$$

subject to

$$\dot{k}_t = f(k_t, l_t) - c_t - m_t - \delta k_t - k_t(b - d) \tag{2}$$

$$\dot{h}_t = g(m_t) - \delta h_t - h_t(b - d) \tag{3}$$

$$\dot{l}_t = (1 - l_t)(b + \gamma(h_t) - \alpha(h_t) l_t) \tag{4}$$

$$k_t, h_t \geq 0, 0 \leq l_t \leq 1, m_t \geq 0 \forall t; \tag{5}$$

$$k_0, h_0, l_0 \text{ given, and } l_0 > 0.$$

The non-concavity

- Note that (4) is non-concave.
- Look at the Hessian:

$$\begin{pmatrix} 2\alpha(h) & -(\gamma'(h)-\alpha'(h)l)-\alpha'(1-l) \\ -(\gamma'(h)-\alpha'(h)l)-\alpha'(1-l) & (1-l)(\gamma''(h)-\alpha''(h)l) \end{pmatrix}$$

- Thus, cannot use Mangasarian sufficiency conditions.
- Difficult to use Arrow sufficiency conditions.

Endemic steady state

Proposition

There exists an endemic steady state ($I^* < 1$) if and only if $b < \overline{\alpha} - \underline{\gamma}$ and there is a solution (I^*, k^*, m^*, h^*) to the following system of equations:

$$
\begin{aligned}
I^*(h^*) &= \frac{\gamma(h^*) + b}{\alpha(h^*)} \\
f_1(k^*, I^*) &= \delta + \theta + b - d \\
g(m^*) &= (\delta + b - d)h^* \\
m^*(f_1(k^*, I^*) &- f_2(k^*, I^*)I'_\theta(h^*)g'(m^*)) = 0 \\
m^* &\geq 0 \\
f_1(k^*, I^*) &\geq f_2(k^*, I^*)I'_\theta(h^*)g'(m^*),
\end{aligned}
$$

where we define $I'_\theta(h^*) := \frac{(1-I^*)(\gamma'(h^*)-\alpha'(h^*)I^*)}{\theta+\alpha(h^*)-b-\gamma(h^*)}$.

Endemic steady state

For endemic steady states, there are two cases:

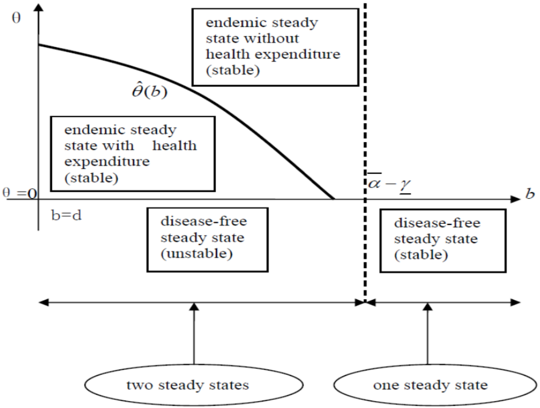
1. $m^* = 0$ (endemic steady state without health expenditure)
2. $m^* > 0$ (endemic steady state with health expenditure)

Endemic steady state without health expenditure ($m^* = 0$) exists iff

$$f_1(\underline{k}, \underline{l}) \geq f_2(\underline{k}, \underline{l}) l'_\theta(0) g'(0),$$

where $l'_\theta(0) := \frac{(1-\underline{l})(\gamma'(0)-\alpha'(0)\underline{l})}{\theta+\overline{\alpha}-b-\underline{\gamma}}$. Otherwise endemic steady state with health expenditure ($m^* > 0$) exists.

Steady state and stability properties of the SIS-ECON growth model



Endemic steady state w/o health expenditures

- For the endemic case w/o health expenditure, linearize the system around the steady state:

$$\mathcal{J}_2 = \begin{pmatrix} \theta & 0 & f_2^* & -1 \\ 0 & -\delta-(b-d) & 0 & 0 \\ 0 & (1-I^*)(\gamma'^* - \alpha'^* I^*) & \overline{\alpha}-(\underline{\gamma}+b) & 0 \\ c^* f_{11}^* & 0 & c^* f_{12}^* & 0 \end{pmatrix}.$$

- The eigenvalues are $\Lambda_1 = -\delta - (b-d) < 0$, $\Lambda_2 = \frac{\theta - \sqrt{\theta^2 - 4c^* f_{11}}}{2} < 0$, $\Lambda_3 = \frac{\theta + \sqrt{\theta^2 - 4c^* f_{11}}}{2} > 0$, and $\Lambda_4 = (\underline{\gamma} + b) - \overline{\alpha} < 0$

- The system is locally saddle stable and has a unique stable path

Human capital formation and impact on infections: externality

- Law of motion for labor supply
  1. Perceived diseases transmission: Each household takes the proportion of the population that is infected (Π) as given and ignores affect of their decisions on aggregate dynamics (Geoffard and Philipson (1996), Gersovitz and Hammer (2004)):

$$\dot{s} = (b+\gamma)(1-s) - \underbrace{\alpha \boldsymbol{\Pi} s}_{true \quad \alpha(1-s)s} .$$

  2. Since labor supply inherits dynamics of s, we have

$$\dot{L} = (b+\gamma)(1-L) - \alpha \boldsymbol{\Pi} L.$$

Control of diseases

- Contact rate is endogenous and depends on effective capital:

$$q = \frac{H}{K}$$

Assumption: Define the effective health capital $q := \frac{H}{K}$. The contact rate $\alpha(q)$ is a C^2 function:

1. $\alpha' < 0$, $\alpha'' > 0$ and $\lim_{q \to 0} \alpha' \to -\infty$, $\lim_{q \to \infty} \alpha' \to 0$;
2. Let $\overline{\alpha}$ and $\underline{\alpha}$ be the upper and lower bound, respectively.

$$\frac{b+\gamma}{\overline{\alpha}} < \frac{\rho - b + d}{\psi} < \frac{b+\gamma}{\underline{\alpha}} < 1.$$

Firms

- Production:
  1. Cobb-Douglas technology for production:

$$Y = AK^{\beta}(euL)^{1-\beta}$$

  2. Profit maximization:

$$\begin{aligned} R &= \beta AK^{\beta-1}(euL)^{1-\beta} \\ W &= (1-\beta)AK^{\beta}(euL)^{-\beta}. \end{aligned}$$

A **competitive equilibrium** is a feasible allocation $\{C, K, H, I_K, I_H, L, u, e\}$ and a price system $\{R, W\}$ such that, given prices:

1. Household solves

$$\max_{\{C, I_K, I_H, u\}} \int_0^\infty e^{-(\rho - b + d)t} u(C) dt,$$

s.t.

$$\begin{aligned}
&\text{Physical capital} && \dot{K} = I_K - \delta K - (b-d)K \\
&\text{Health capital} && \dot{H} = I_H - \delta H - (b-d)H \\
&\text{Human capital} && \dot{e} = \psi e L (1-u) \\
&\text{Effective labor} && \dot{L} = (b+\gamma)(1-L) - \alpha \left(\frac{H}{K}\right) \Pi L \\
&\text{Budget} && C + I_K + I_H = RK + WeuL \\
& && 0 \le u \le 1,\ 0 \le L \le 1,\ I_H \ge 0, \text{ with } e_0, K_0, H_0 \text{ and } L_0 \text{given.}
\end{aligned}$$

2. Firms maximize profits: $Y = AK^\beta (euL)^{1-\beta}$
3. Capita, labor and goods markets clear;
4. Since each household is representative of the population, in equilibrium

$$\Pi = 1 - L.$$

Competitive Equilibrium

1. Disease-free BGP:

$$L^* = 1, \quad u^* = \frac{\rho - b + d}{\psi}, \quad \text{and} \quad g = \psi - (\rho - b + d);$$

2. Disease-endemic BGP:

$$L^* = \frac{b + \gamma}{\alpha(q^*)}, \quad u^* = \frac{\rho - b + d}{\psi L^*}, \quad \text{and} \quad g = \psi L^* - (\rho - b + d)$$

3. Disease-endemic poverty trap.

$$L^* = \frac{b + \gamma}{\alpha(q^*)}, \quad u^* = 1, \quad \text{and} \quad g = 0$$

- The driving force for positive growth is time spent for human capital accumulation ($u^* < 1$).

First wave of models...

- In the same line as GH, the first Covid-driven epi-econ models published in Spring/Summer 2020 have focused on central planner problems where anauthority has to choose an appropriate epidemic control policy (lockdown timing and intensity, testing and vaccination strategies, mix-strategies,..etc) to maximize a social welfare function accounting both for the economic cost and human lives losses under implicit or explicit health system capacity constraints.
- Examples of such an approach can be found in Alvarez et al. (2022), Piguillem and Shi (2022), Aspri et al. (2021) or Caulkins et al. (2021). Reviews of this first wave of papers can be found in Boucekkine et al. (2021) and Amir and Boucekkine (2022).
- The human populations considered in these early models are largely homogenous with the exception of some papers explicitly integrating (discrete) age structures like Acemoglu et al. (2022) or Gollier (2020). Fabbri et al. (2021) consider a continuous age structure in the tradition of the seminal Kermac-McKendrick model.

...with already some variations

- More in line with a branch of math epidemiology, some authors have resorted to infection dynamics through infective links within given (exogenous) social networks: e.g. Freiberger et al. (2022), Fajgelbaum et al. (2021), Bisin and Moro (2022) or Mandel and Bouveret (2021). Geographic epi-econ models are equally proposed (LaTorre et al. , 2021, or Rothert, 2022)
- Other authors used matching models: e.g Camera and Gioffré (2021) have constructed a frame based on the theory of random matching expliciting how epidemics spread through economic activity and how lockdowns impact the contagion process and social welfare.
- Stochastic central planning problems have also been treated: e.g Federico and Ferrari (2021) have determined the optimal lockdown policy under a stochastic epidemic diffusion process.
- Also, the supply chain discruptions due to Covid and lockdown are considered in several contributions: e.g. Bodenstein et al. (2022) have combined an epidemiological model, calibrated to capture the spread of the COVID-19 virus, with a multisector model, designed to capture key characteristics of the U.S. Input Output Tables

Missing: Individuals behaviors in response to epidemics

- The vast majority of the epi-econ models of the first generation have in common to analyze optimal epidemic control policies **for given (homogenous) population behavior**, therefore incurring in a kind of Lucas critique case.
- As recently outlined by Bisin and Moro (2022), such a treatment may lead to considerable biases, which in turn could generate misleading epidemic control policy responses. They numerically illustrate this potential problem by comparing a network-based model of epidemic diffusion with and without individual behavior.
- Individual behavior here is non-strategic, it mimics previous work by Bouveret and Mandel (2021) where agents in any node of the network invests resources to decrease the infectivity of their link. This is indeed a very reduced-form of individual behavior.
- This simple exercise show how urgent is the elaboration of a new generation of hybrid epi-econ models for a reliable design of epidemic control policies to deal with emerging diseases.

Some epi-econ models with endogenous individual behavior

- In addition to Bisin and Moro (2022), Allcott et al (2021) is a nice illustration of this stream, in particular as to heterogenous response among individuals, here parametrized by their partisan preferences. While this heterogenous behavior is perfectly rationalized by the stylized epi-econ model constructed, it has no strategic ingredient (no game).
- Very few papers include endogenous individual behavior AND strategic interactions, among them Toxvaerd (2020) who provides with a game-based theory of "equilibrium social distancing" and Baril-Tremblay et al. (2021) on "equilibrium self-isolation".
- However, the games considered are actually static, which incidentally also means that these models are not really epi-econ models in that the dynamic diffusion of the outbreak is omitted.
- Elaborating dynamic versions (could be also sequential games) notably between the population and the health authority is an interesting avenue. Mimics similar problems with asymmetric information and incomplete control (e.g. inflation).

Model outlines (Caulkins et al., 2021)

- $\gamma(t)$ is the actual number of people working as a proportion of those who would normally be working, so apart from COVID-19 we would have $\gamma(t) = 1$. As soon as the lockdown starts, $\gamma(t)$ will drop below 1, which hurts the economy.
- Interestingly $\gamma(t)$ is modeled as a state variable, not a control. First, policy makers do not get to choose directly the level of employment. Second, adjusting the level of employment takes time and is costly. If a country that has shut down its auto manufacturing permits that supply chain to reopen, it will take time to reestablish connections.

$$\dot{\gamma}(t) = u(t).$$

- $z(t)$ captures "lockdown fatigue" through a state equation driven by the rate of COVID-induced unemployment.

$$\dot{z}(t) = \kappa_1 (1 - \gamma(t)) - \kappa_2 z(t).$$

where κ_1 governs the rate of fatigue accumulation, κ_2 is its rate of decay.

State dynamics

The full state dynamics can be written as

$$
\begin{aligned}
&\dot{S}(t) = \nu N(t) - \beta(\gamma(t), z(t))\frac{S(t)I(t)}{N(t)} - \mu S(t) + \varphi R(t) \\
&\dot{I}(t) = \beta(\gamma(t))\frac{S(t)I(t)}{N(t)} - (\alpha + \mu + \mu_I)I(t) \\
&\dot{R}(t) = \alpha I(t) - \mu R(t) - \varphi R(t) \\
&\dot{\gamma}(t) = u(t), \quad \gamma(0) = 1 \\
&\dot{z}(t) = \kappa_1(1 - \gamma(t)) - \kappa_2 z(t), \quad z(0) = 0 \\
&\gamma(t) \leq 1, \quad 0 \leq t \leq T \\
&\beta(\gamma(t), z(t))\beta_1 + \beta_2\left(\gamma(t)^\theta + f\frac{\kappa_2}{\kappa_1}z(t)(1 - \gamma(t)^\theta)\right)
\end{aligned}
$$

where $N(t) = S(t) + I(t) + R(t)$ is the total population.

The objective function

- The cost of epidemic control has 2 components. One is health cost: here, it depends on the number of infected probably requiring intensive care (prob=p) given capacity constraints (H_{max}).

$$V_h(I,\gamma) \equiv M\left(\xi_1 pI(t) + \xi_2 maxs(\{0, pI(t) - H_{\max}\}, \zeta)\right)$$

$$with \;\; maxs(\{0, pI(t) - H_{\max}\}, \zeta) \equiv \frac{1}{\zeta} \log\left(1 + ^{\zeta(pI(t) - H_{\max})}\right), \quad \zeta \gg 1.$$

- Adding the economic costs, the objective function looks like

$$V \equiv \int_0^T \left(V_l(L(t),\gamma(t)) - V_h(I(t),\gamma(t)) - V_u(u(t),\gamma(t))\right) + \ldots.$$

with $V_l(L(t),\gamma(t)) = K\gamma(t)^\sigma L(t)^\sigma$, and

$$V_u(u(t),\gamma(t)) = \begin{cases} c_l u(t)^2 & u(t) \leq 0 \\ c_r(z(t)+1)u(t)^2 & u(t) > 0 \end{cases}$$

A rich and mostly open research program

1. Just like other disciplines (starting with epidemiology and microbiology), economists have been rocked by the Covid-19 shock, a singular epidemic in many respects.
2. The first wave of research papers has in a way taken the approach opened by Gersovitz and Hammer: central planner problems internalizing the externalities generated by the infection dynamics.
3. However, the specificities of the Covid-19 outbreak and inherent inherent control policies (the so-called Non-Pharmaceutical Interventions) have induced much more interesting and intricate research questions than in GH.
4. This said, as outlined in the slide above, the hardest questions are still wide open. We shall examine some of the them in the next lecture.