Lecture 2:

Communication & Cooperation

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Introduction

- Intuitive idea: communication is necessary for cooperation

Antitrust Law

- Tacit vs. explicit collusion
  - "Tacit collusion … describes a process, not in itself unlawful, by which …firms in a concentrated market might in effect share monopoly power, setting their prices at a profit-maximizing supracompetitive level …" (US Supreme Court, 1993)
- Theoretical basis for the distinction?

Cartel meetings

Industry	Frequency of meetings
Choline chloride	
Citric acid	
Copper tubes	
Lysine	?
Plasterboard	
Vitamins A and E	
Zinc phosphate	

Cartel meetings

Industry	Frequency of meetings
Choline chloride	2-3 weeks
Citric acid	monthly
Copper tubes	1-2 months
Lysine	monthly
Plasterboard	quarterly
Vitamins A and E	weekly/quarterly
Zinc phosphate	monthly

Goal

- To develop tools and methodology to study when cooperation/collusion is facilitated by
- Direct (face-to-face) communication
- Indirect (via a third-party) communication

Example 1

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

Claim: With **perfect** monitoring, $\forall \delta \geq 1/3$, there is an efficient equilibrium.

Imperfect private monitoring

	C	D
C	2, 2	-1, 3
D	3 -1	0, 0

	h	l
h	½	0
l	0	½

	h	l
h	¼	¼
l	¼	¼

Players only see private signals h or l after every period.

Imperfect private monitoring

	C	*D*
C	2, 2	-1, 3
D	3 -1	0, 0

	h	*l*
h	½	0
l	0	½

	h	*l*
h	¼	¼
l	¼	¼

Players only see private signals *h* or *l* after every period.

Private monitoring precludes cooperation

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	½	0
l	0	½

	h	l
h	¼	¼
l	¼	¼

Claim: $\forall\delta$, DD is the only eqm path.

Permanent deviation to D leaves marginal distribution on other's signals unchanged.

Communication restores cooperation

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	½	0
l	0	½

	h	l
h	¼	¼
l	¼	¼

CHEAP TALK

Claim: $\forall \delta \geq ½$, CC is an eqm path w/comm.

Play C and report truthfully; if reports match, play C; otherwise, play D forever.

Example 1

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	½	0
l	0	½

	h	l
h	¼	¼
l	¼	¼

Example 2

	C	*D*
C	2, 2	-1, 3
D	3, -1	0, 0

	h	*l*
h	¼	¼
l	¼	¼

	h	*l*
h	½	0
l	0	½

Example 2

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	¼	¼
l	¼	¼

	h	l
h	½	0
l	0	½

Communication $\not\Rightarrow$ cooperation

	h	l
h	¼	¼
l	¼	¼

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	½	0
l	0	½

Claim: $\forall \delta \geq \frac{1}{2}$, only *DD* is played w/ or w/o comm.

Communication $\not\Rightarrow$ cooperation

	h	l
h	¼	¼
l	¼	¼

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

	h	l
h	½	0
l	0	½

Play D and lie ("h" and "l" with prob. ½). Joint dist. over P1's reports and P2's signals is unchanged.

Example 3

	R	P	S
R	10, 10	0, 11	11, 0
P	11, 0	10, 10	0, 11
S	0, 11	11, 0	10,10

$\frac{1}{3}$	0	0
0	$\frac{1}{3}$	0
0	0	$\frac{1}{3}$

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

π Communication is not necessary for cooperation

	R	P	S
R	10, 10	0, 11	11, 0
P	11, 0	10, 10	0, 11
S	0, 11	11, 0	10,10

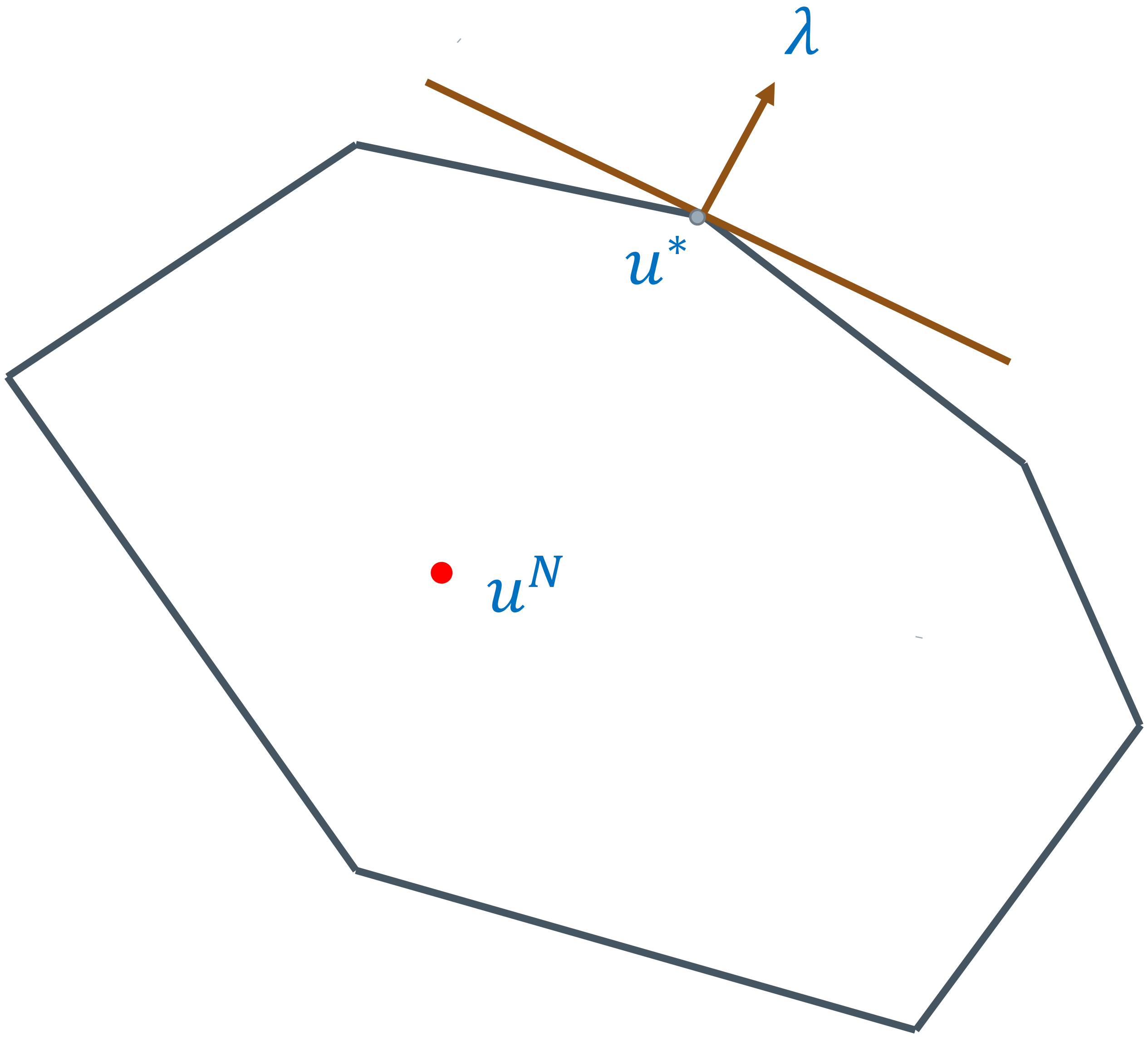
$\frac{1}{3}$	0	0
0	$\frac{1}{3}$	0
0	0	$\frac{1}{3}$

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Claim: $\forall \delta \geq \frac{3}{11}$, there is an efficient eqm w/o comm.

One-shot game

- n players
- Finite actions A_i
- Payoffs $u_i: A \rightarrow \Re$
- Nash equilibrium a^N
- Efficient action $a^*: \max_a \lambda \cdot u(a)$
- a^* not a Nash equilibrium and $u(a^*) \gg u(a^N)$


λ
u*
u^N

Repeated game

- Actions a_i
- Signals $y_i \in Y_i$
- Joint actions generate signals as $q(\cdot\,|a) \in \Delta(Y)$
- Observe private signal y_i only

- Strategy maps own past actions and own past signals to current actions

Repeated game (Example: Stigler 1964)

- Actions a_i (= **Prices** p_i)
- Signals $y_i \in Y_i$ (= **Sales**)
- Joint actions (**all prices**) generate signals (**sales**) $q(\cdot|a) \in \Delta(Y)$
  - **Noisy demand**
- Observe private signal y_i only (**sales are private**)

- Strategy maps own past actions (**prices**) and own past signals (**sales**) to current actions
- (**Ex post profits = own price** $\times$ **own sales**)

Repeated game with communication

- Actions a_i
- Signals $y_i \in Y_i$
- Actions generate signals as $q(\cdot\,|a) \in \Delta(Y)$
- Observe private signal y_i only
- Exchange cheap talk messages
- Strategy maps own past actions, own past signals and all past messages to current actions

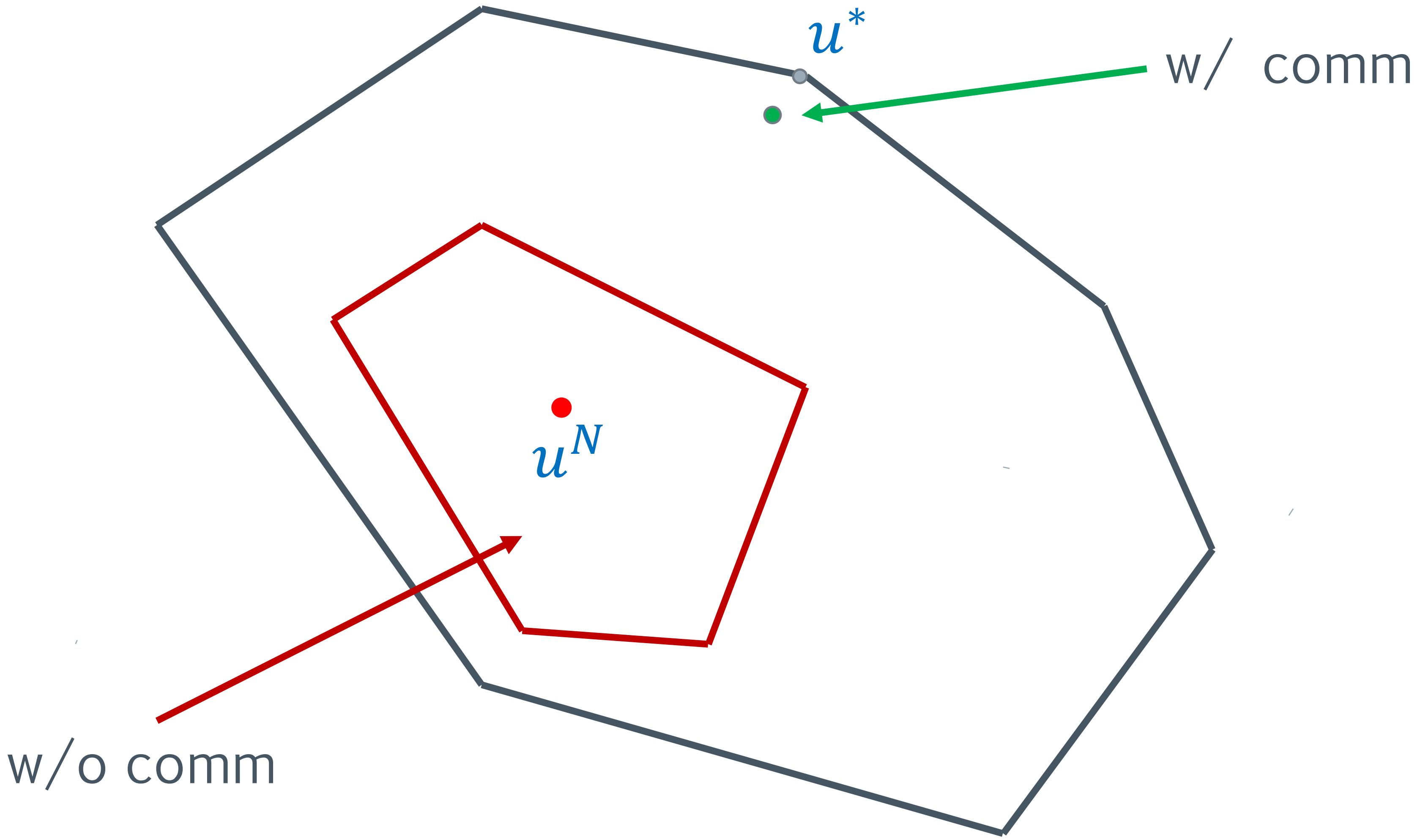
Communication is necessary for cooperation

Main result: Fix any large δ. Under certain conditions on the monitoring, there is a nearly efficient eqm w/ comm whereas all eqa w/o comm are bounded away from eff.

Step 1: bound on payoffs w/o comm

Step 2: construct a nearly efficient eqm w/comm

Not a folk-theorem!



u^*
w/ comm
u^N
w/o comm

Background

Folk Theorem with communication:

Compte (*Ecta*, 1998), Kandori & Matsushima (*Ecta*, 1998)

Folk Theorem without communication:

Sugaya (2013)

Necessity of communication:

Awaya & Krishna (*AER*, 2016)

Step 1: Bound without communication

Bound depends on

- Payoffs: u
- Monitoring quality:

$$\eta = \max_i \max_{a,b_i} \left\| q_{-i}\left(\cdot \mid b_i, a_{-i}\right) - q_{-i}\left(\cdot \mid a\right) \right\|_{TV}$$

- Discount factor: δ

Step 1: Bound without communication

Proposition:

$$NE\left(G_{\delta,q}\right) \subseteq \varepsilon\text{-}CCE(G)$$

where $\varepsilon = 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$.

Step 1: Bound without communication

Proposition:

One-shot game

$$NE(G_{\delta,q}) \subseteq \varepsilon\text{-}CCE(G)$$

Repeated game

where $\varepsilon = 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$

Step 1: Bound without communication

Proposition:

$$NE(G_{\delta,q}) \subseteq \varepsilon\text{-}CCE(G)$$

Coarse correlated equilibria

where $\varepsilon = 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$.

Monitoring quality

Coarse Correlated Equilibrium

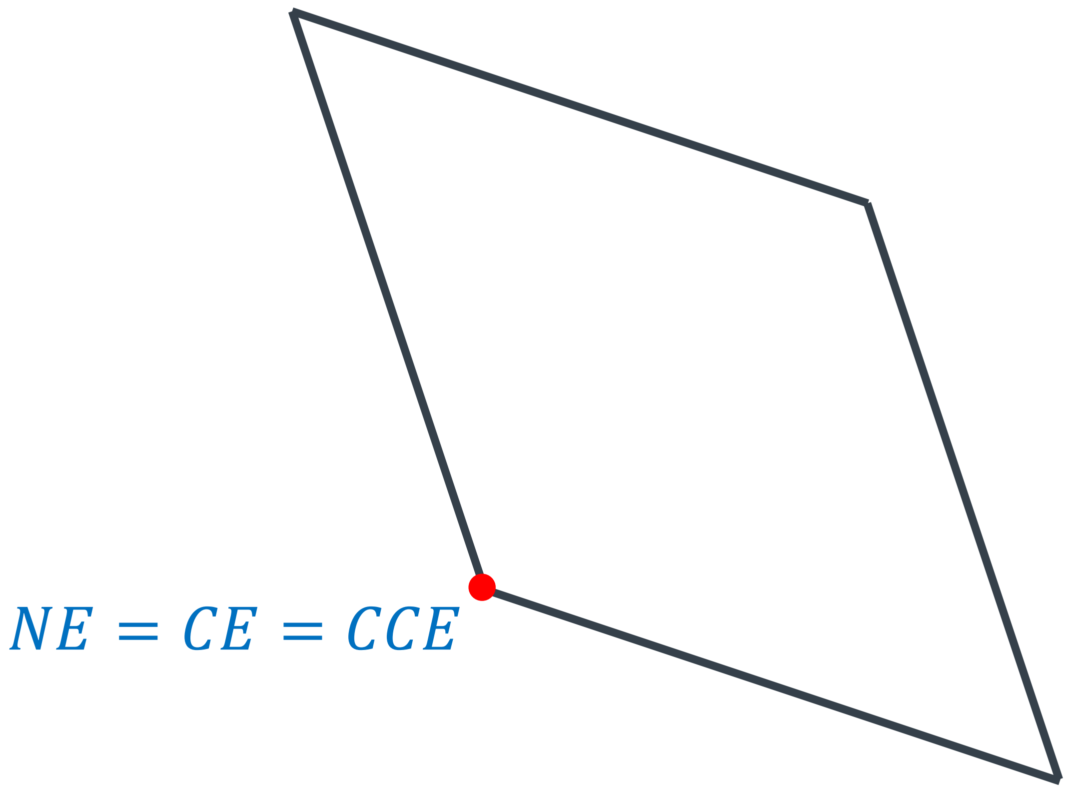
$\beta \in \Delta(\times_{j \in N} A_j)$ is a coarse correlated equilibrium if for all i and $a_i \in A_i$,

$$u_i(\beta) \geq u_i(a_i, \beta_{-i})$$

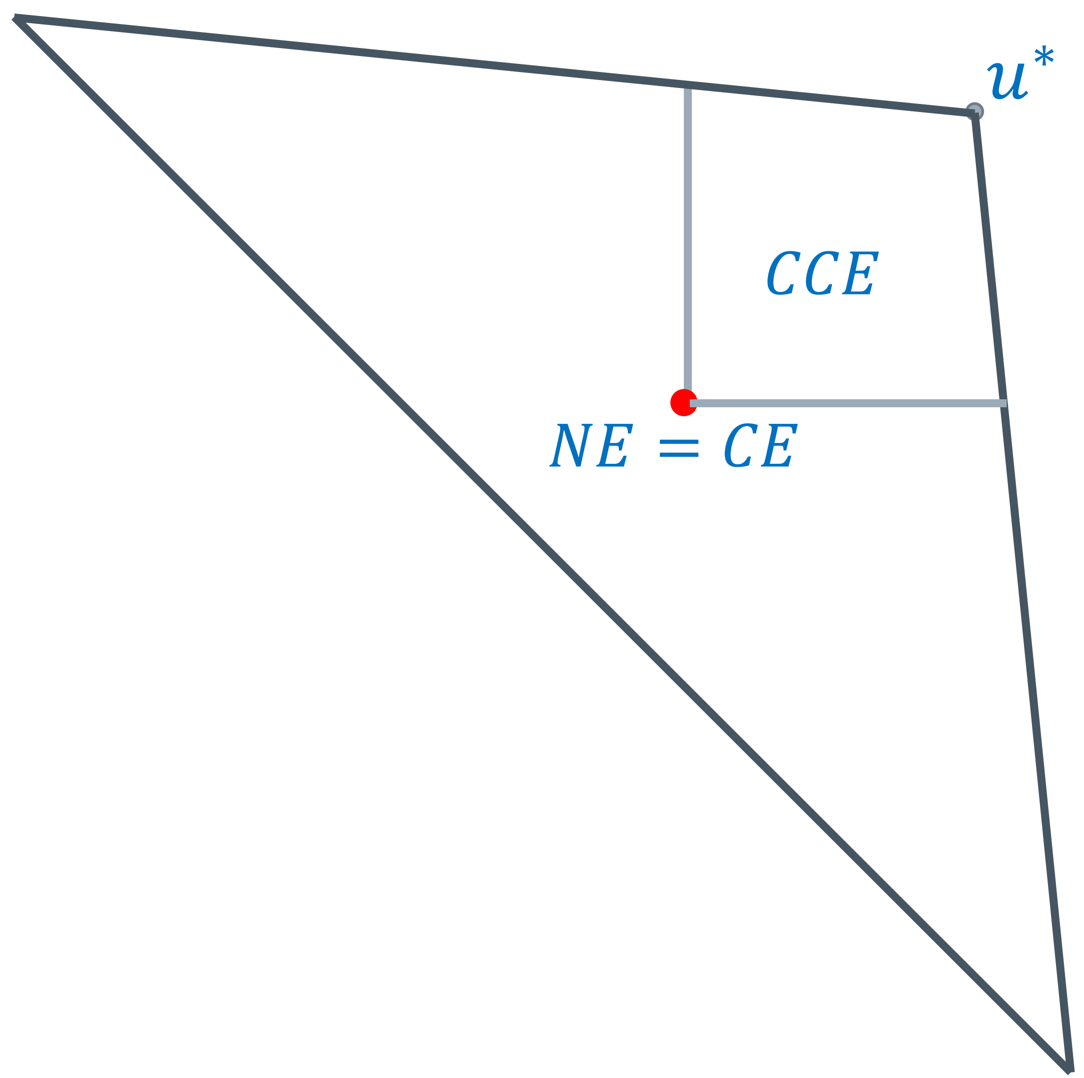
Moulin & Vial (1979); Roughgarden (2016)

$$NE \subset CE \subset CCE$$

Prisoners' Dilemma



R-P-S

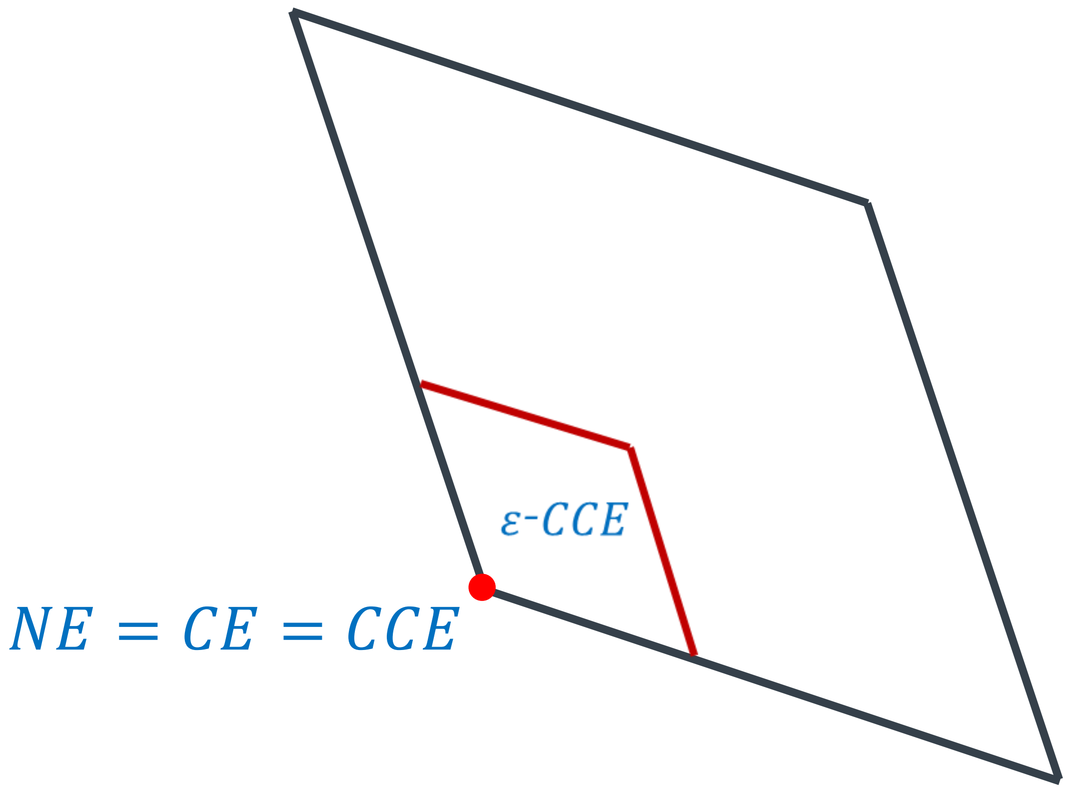


ε-Coarse Correlated Equilibrium

$\beta \in \Delta(\times_{j \in N} A_j)$ is an ε-coarse correlated equilibrium if for all i and $a_i \in A_i$,

$$u_i(\beta) \geq u_i(a_i, \beta_{-i}) - \varepsilon$$

Prisoners' Dilemma



Static incentives

For $v \in F$ define:

$$\Phi(v) = \min_{\beta \in \Delta(A)} \max_i \max_{a_i} u_i(a_i, \beta_{-i}) - u_i(\beta)$$
$$\text{s.t.} \quad u(\beta) = v$$

- Φ is convex
- $\Phi(v) \leq \varepsilon \Leftrightarrow \varepsilon\text{-}CCE(G)$

Dynamic incentives

Lemma 1:

$$\Phi(v) = \min_{\alpha \in \Delta(A)^\infty} \max_i \max_{\bar{\sigma}_i} v_i(\bar{\sigma}_i, \alpha_{-i}) - v_i(\alpha)$$

$$\text{s.t.} \quad v(\alpha) = v$$

Proof:

Dynamic smoothing of incentives (Φ is convex).

Step 1: Bound without communication

- Let σ be a strategy with payoffs $v(\sigma)$ such that

$$v \equiv v(\sigma) \notin \varepsilon\text{−}CCE(G)$$

- Then $\Phi(v) > \varepsilon$
- $\exists i$ and $\overline{\sigma}_i$ such that $v_i(\overline{\sigma}_i, \alpha_{-i}) - v_i(\sigma) > \varepsilon$

Step 1: Bound without communication

$$v_i(\overline{\sigma}_i, \sigma_{-i}) - v_i(\sigma) =$$

$$v_i(\overline{\sigma}_i, \sigma_{-i}) - v_i(\overline{\sigma}_i, \alpha_{-i}) + v_i(\overline{\sigma}_i, \alpha_{-i}) - v_i(\sigma)$$

$\uparrow$ Loss when punished $\qquad$ $\uparrow$ Gain when unpunished

Step 1: Bound without communication

Lemma 2:

$$|v_i(\overline{\sigma}_i,\ \sigma_{-i}) - v_i(\overline{\sigma}_i,\ \alpha_{-i})| < 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$$

η is monitoring quality

$\bar{u}$ is largest payoff

Step 1: Bound without communication

$$v_i(\overline{\sigma}_i, \sigma_{-i}) - v_i(\sigma) =$$

$$v_i(\overline{\sigma}_i, \sigma_{-i}) - v_i(\overline{\sigma}_i, \alpha_{-i}) + v_i(\overline{\sigma}_i, \alpha_{-i}) - v_i(\sigma)$$

$$\vee \qquad\qquad\qquad\qquad \vee$$

$$-2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty \qquad\qquad \varepsilon$$

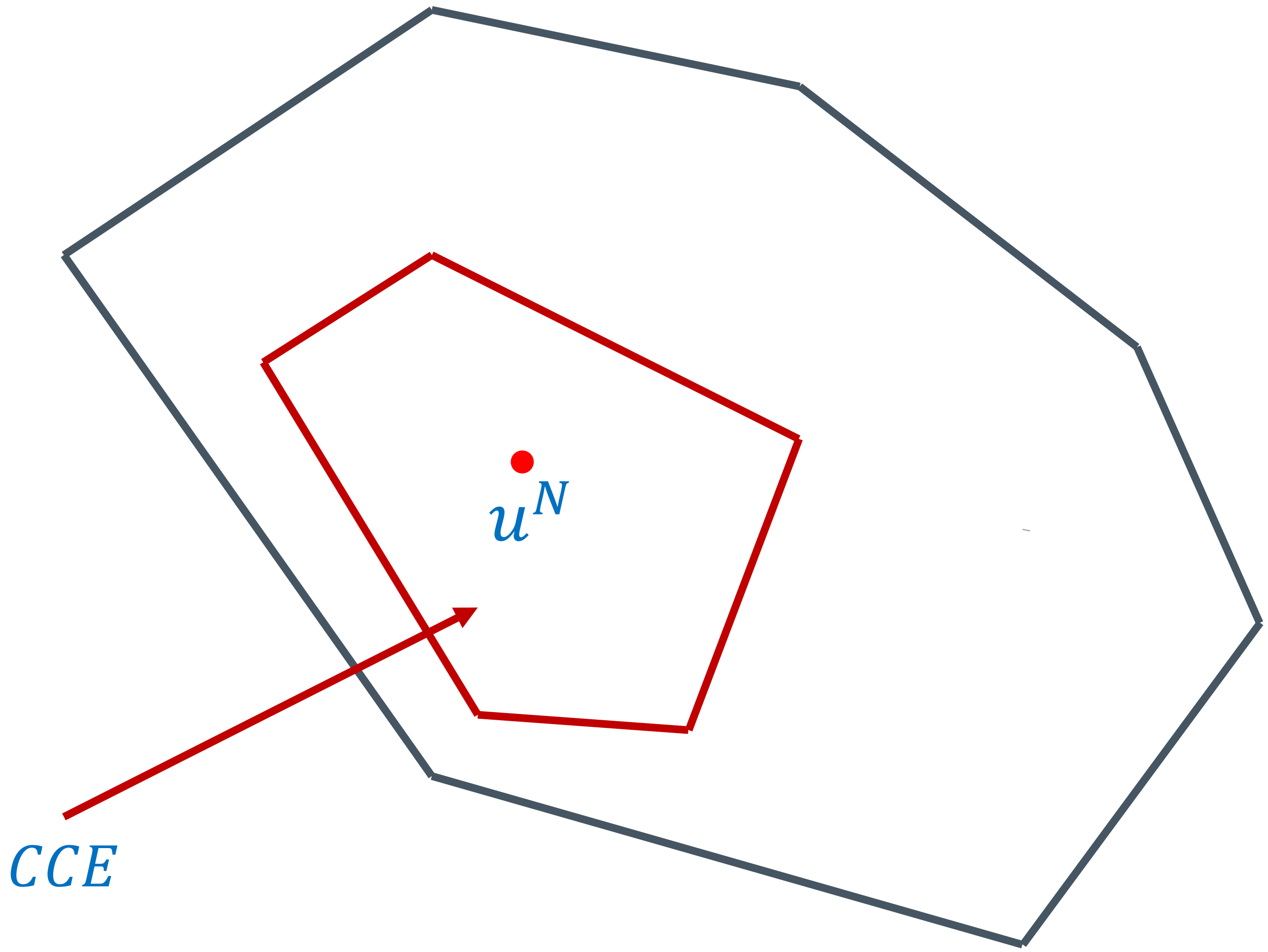
(Lemma 2) (Lemma 1)

Step 1: Bound without communication

Proposition:

$$NE\left(G_{\delta,q}\right) \subseteq \varepsilon\text{-}CCE(G)$$

where $\varepsilon = 2\frac{\delta^2}{1-\delta}\eta \times \|u\|_\infty$.



u^N
CCE

Effective bound without communication

Condition 1: $\nexists$ an efficient coarse correlated eqm.

Ensures bound is effective when $\eta = 0$.

Fails in Example 3 (RPS)

Step 2: Equilibrium with communication

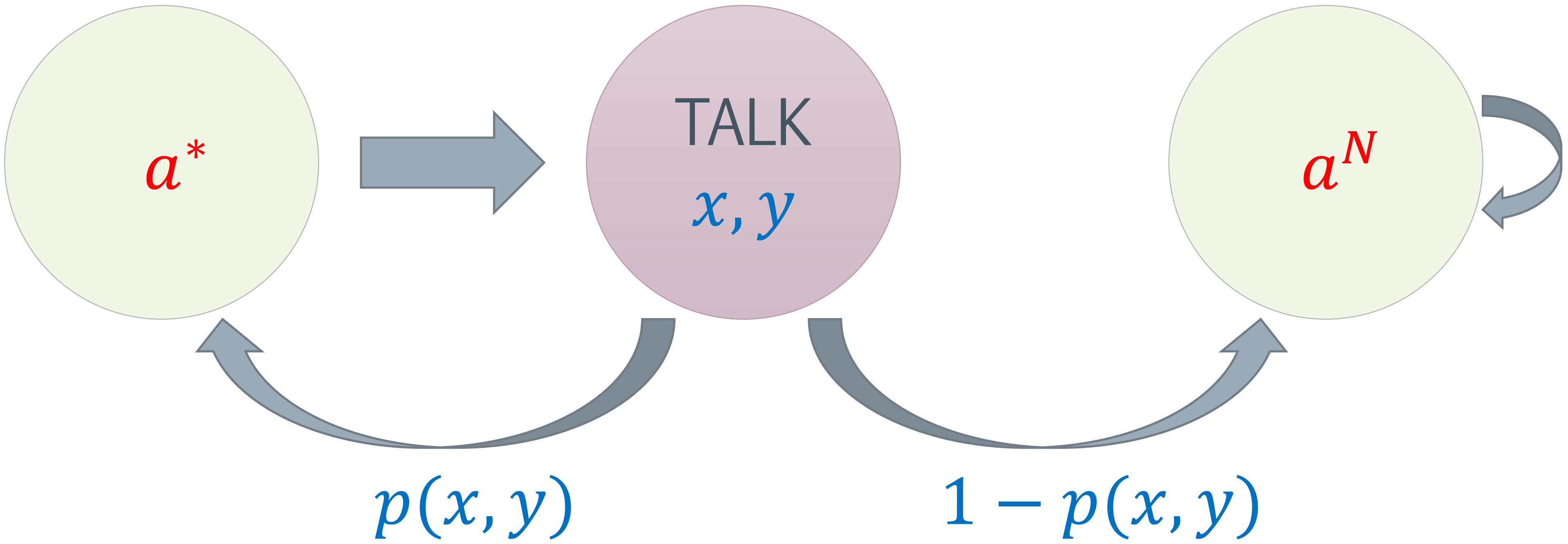
Strategy:

› In state a^* play a_i^*; otherwise, play a_i^N

› If played a_i^*, report signal truthfully; otherwise optimize

› State transitions depend on signal reports x and y

Equilibrium with communication



Truth-telling

Proposition: If types are strictly affiliated, there exists a strictly IC mechanism in which every player has the same **ex post** payoffs.

Truth-telling

Proposition: With strictly affiliated types, there exists a strictly IC mechanism in which every player has the same **ex post** payoffs.

Truth-telling

Suppose q^* is strictly affiliated over $[0,1]^2$. Define

$$P(x,y) = xy - \int_0^x E[Y|X = s]ds - \int_0^y E[X|Y = t]dt$$

Truth-telling

Suppose q^* is strictly affiliated over $[0,1]^2$. Define

$$P(x,y) = xy - \int_0^x E[Y|X=s]ds - \int_0^y E[X|Y=t]dt$$

Claim: q^* is a **strict** correlated eqm of (P,P).

Proof: Report z when signal is x

$$E[P(z,Y)\,|X=x] = zE[Y|X=x] - \int_0^z E[Y|X=s]ds - C(x)$$

First-order condition

$$0 = E[Y|X=x] - E[Y|X=z]$$

Correlation

- Normalize P to $p \in [0,1]$.
- Let $\gamma = \left\| q(\cdot|a^*) - q^0 \right\|_{TV}$ where q^0 is closest perfectly correlated distribution.
- As $\gamma \to 0$, $p(x,y) \to p^0(x,y) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$
- p^0 leads to full efficiency
- Find $\underline{\delta}$ so that eqm with p^0

Main result

Theorem: Fix $\delta > \underline{\delta}$.

There exist $(\bar{\eta}, \bar{\gamma})$ such that for all q with $(\eta, \gamma) \ll (\bar{\eta}, \bar{\gamma})$, there is an eqm w/ comm whose payoffs exceed those in any eqm w/o comm.

η = monitoring quality

γ = correlation in q^*

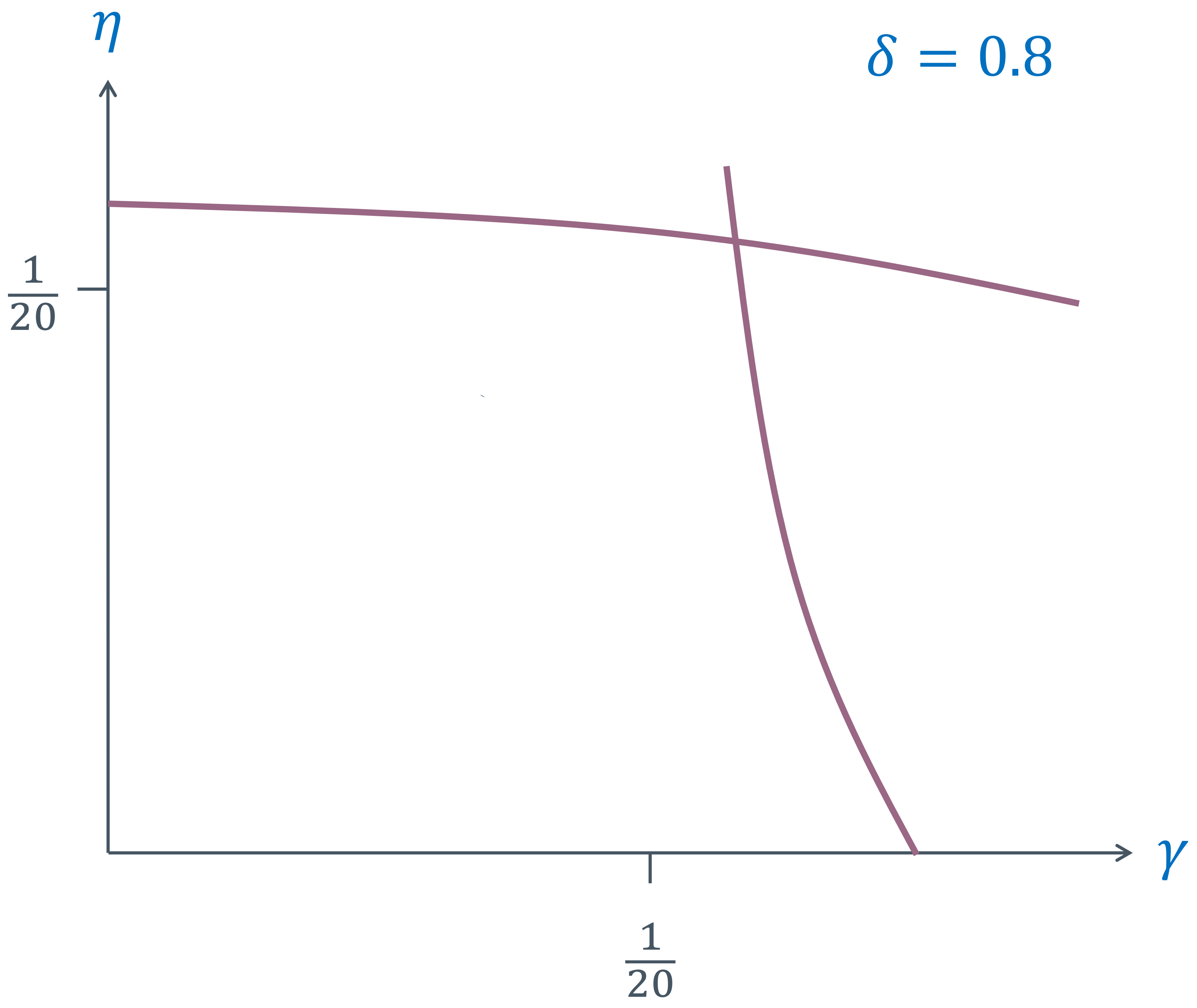
Example 4

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

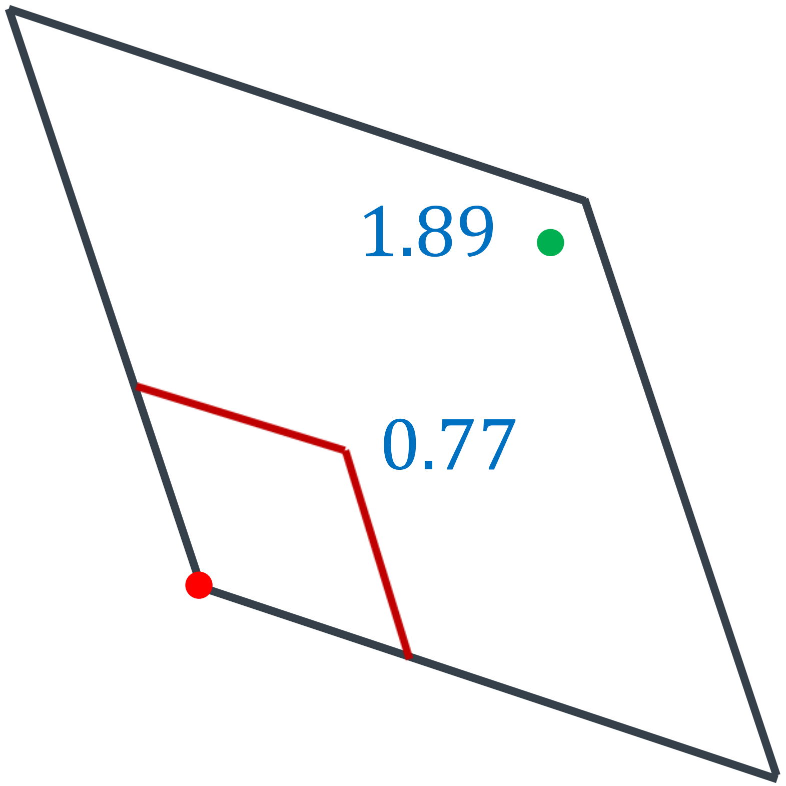
$\frac{1}{3}(1-\gamma)$	$\frac{1}{6}\gamma$	$\frac{1}{6}\gamma$
$\frac{1}{6}\gamma$	$\frac{1}{3}(1-\gamma)$	$\frac{1}{6}\gamma$
$\frac{1}{6}\gamma$	$\frac{1}{6}\gamma$	$\frac{1}{3}(1-\gamma)$

$\left(\frac{1}{3}-\eta\right)^2$	$\frac{1}{9}-\eta^2$	$\frac{1}{9}-\frac{1}{3}\eta$
$\frac{1}{9}-\eta^2$	$\left(\frac{1}{3}+\eta\right)^2$	$\frac{1}{9}+\frac{1}{3}\eta$
$\frac{1}{9}-\frac{1}{3}\eta$	$\frac{1}{9}+\frac{1}{3}\eta$	$\frac{1}{9}$

$\delta = 0.8$



Example 4



$\delta = 0.8$

$\eta = 0.02$
$\gamma = 0.02$

Application to Stigler (1964)

› Repeated oligopoly
  – Firms compete in prices (differentiated goods)
  – Prices and market shocks determine sales

› Compare two scenarios
  1. Firms observe only own prices and sales (secret price cuts)
  2. Firms can communicate sales (cheap talk)

Application to Stigler (1964)

- Two firms
- prices $\boldsymbol{p} = (p_1, p_2) \implies \boldsymbol{Y} = (Y_1, Y_2)$ sales
- $\boldsymbol{Y} \sim f = \log \mathcal{N}\big(\boldsymbol{\mu}(\boldsymbol{p}), \Sigma(\boldsymbol{p})\big)$

Expected Sales

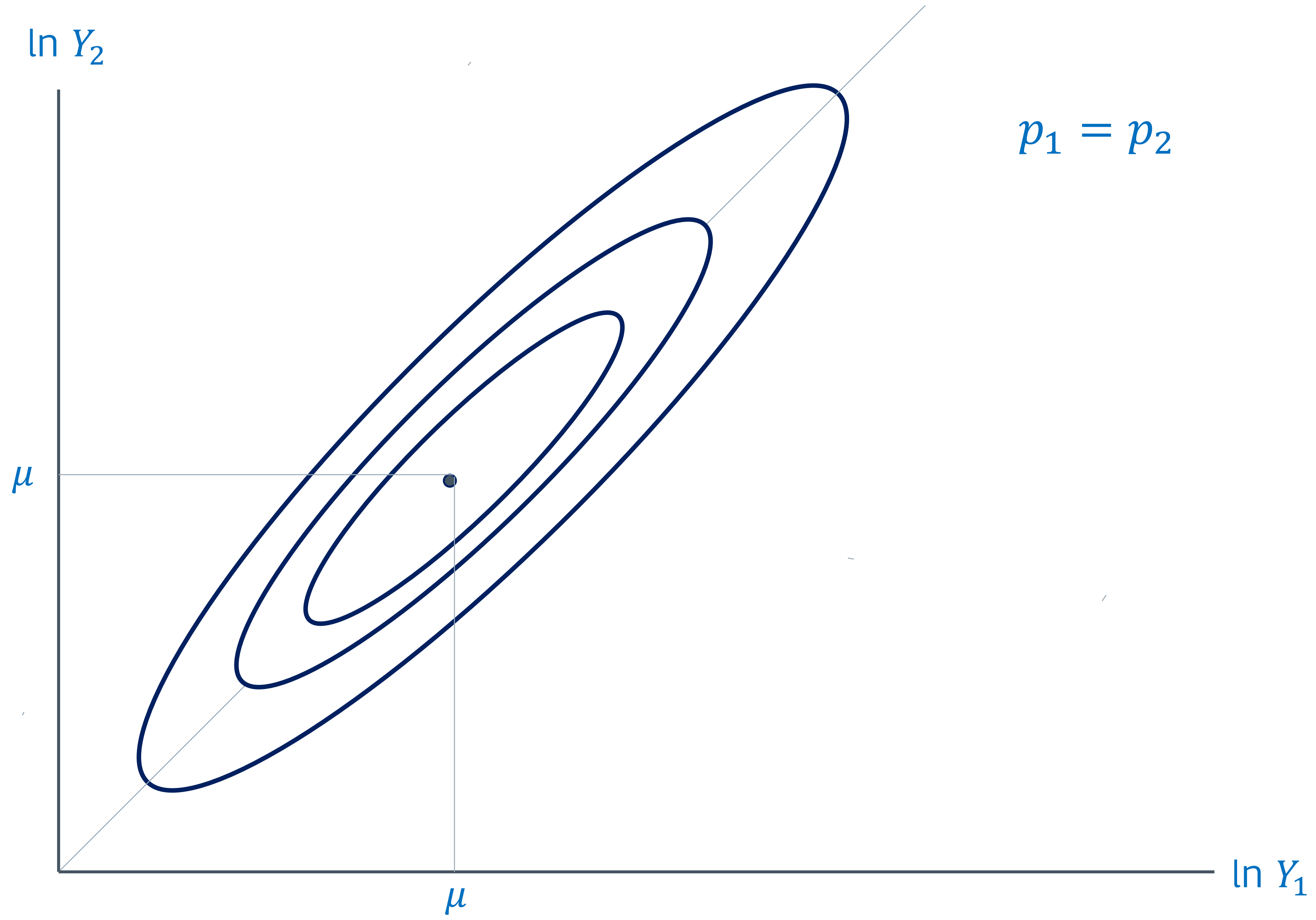
- Expected sales $\mathbb{E}[\,Y_i \mid \boldsymbol{p}]$ ↓ in p_i and ↑ in p_j
  - Example $\mathbb{E}[\,Y_i \mid \boldsymbol{p}] = \alpha - \beta p_i + \ p_j$
- Own price effect > cross price effect
- Expected profits $\pi_i(\boldsymbol{p}) = p_i\, \mathbb{E}[\,Y_i \mid \boldsymbol{p}]$

Correlation in Sales

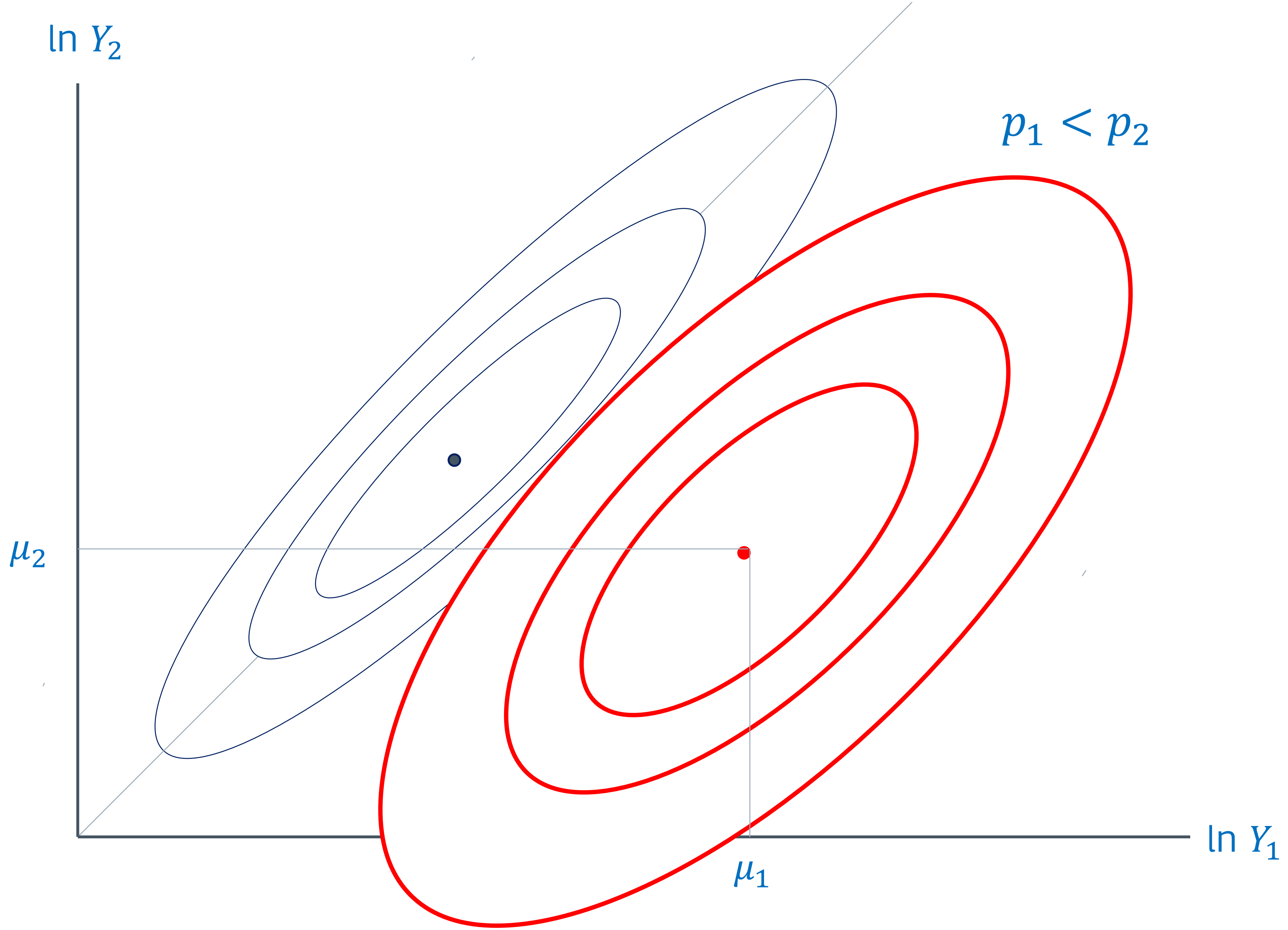
$$\Sigma(\boldsymbol{p}) = \sigma^2 \begin{bmatrix} 1 & \rho(\boldsymbol{p}) \\ \rho(\boldsymbol{p}) & 1 \end{bmatrix}$$

$$\rho(\boldsymbol{p}) > 0$$

$\rho \downarrow$ in price gap $|p_1 - p_2|$



$\ln Y_2$
$p_1 = p_2$
μ
μ
$\ln Y_1$



$\ln Y_2$
$p_1 < p_2$
μ_2
$\ln Y_1$
μ_1

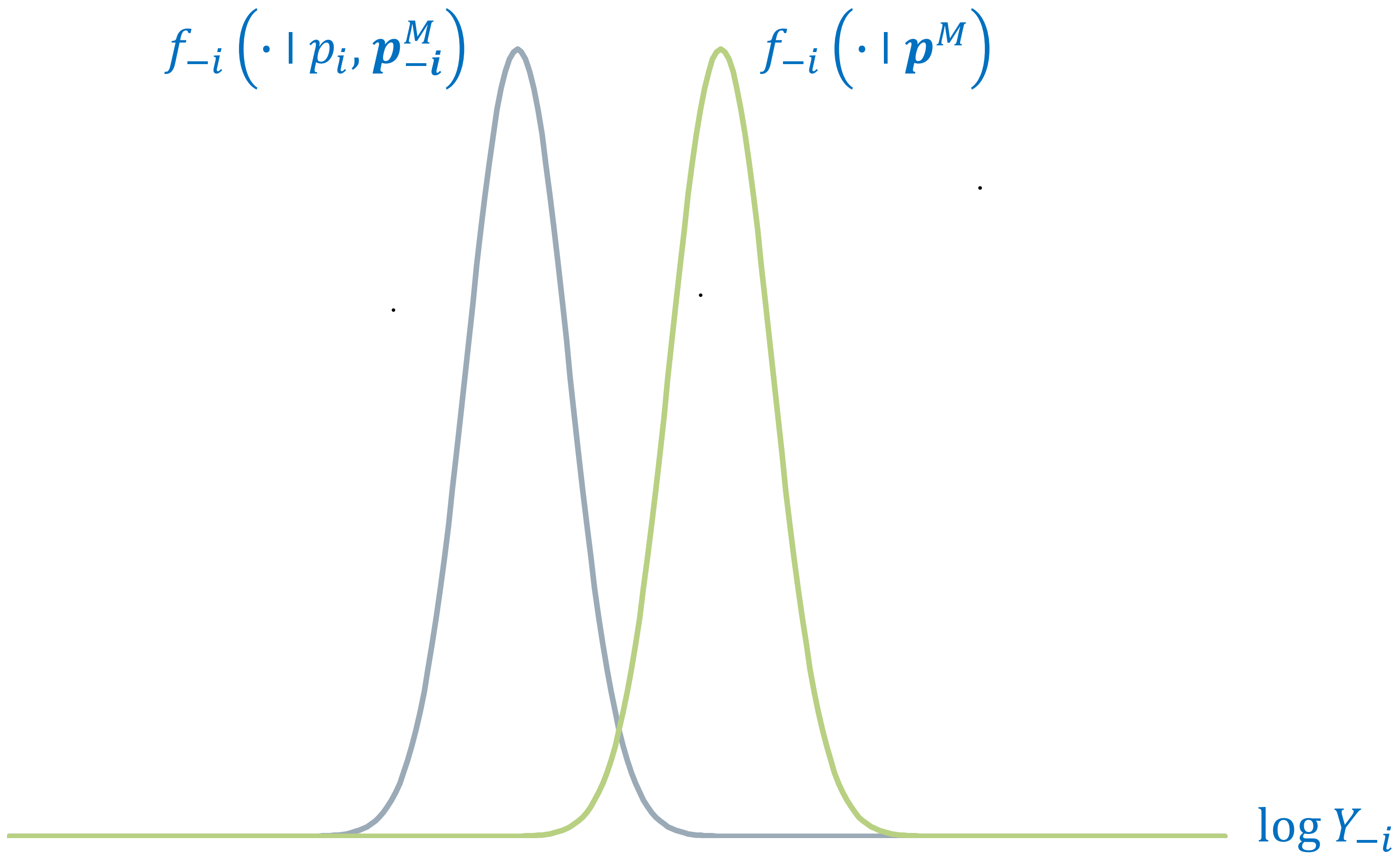
Why is $\rho \uparrow$ in price gap $|p_1 - p_2|$?

› Hotelling model with random transport costs

	high	low
$p_1 = p_2$	$Y_1 = Y_2$	$Y_1 = Y_2$
$p_1 < p_2$	$Y_1 = Y_2$	$Y_1 < Y_2$

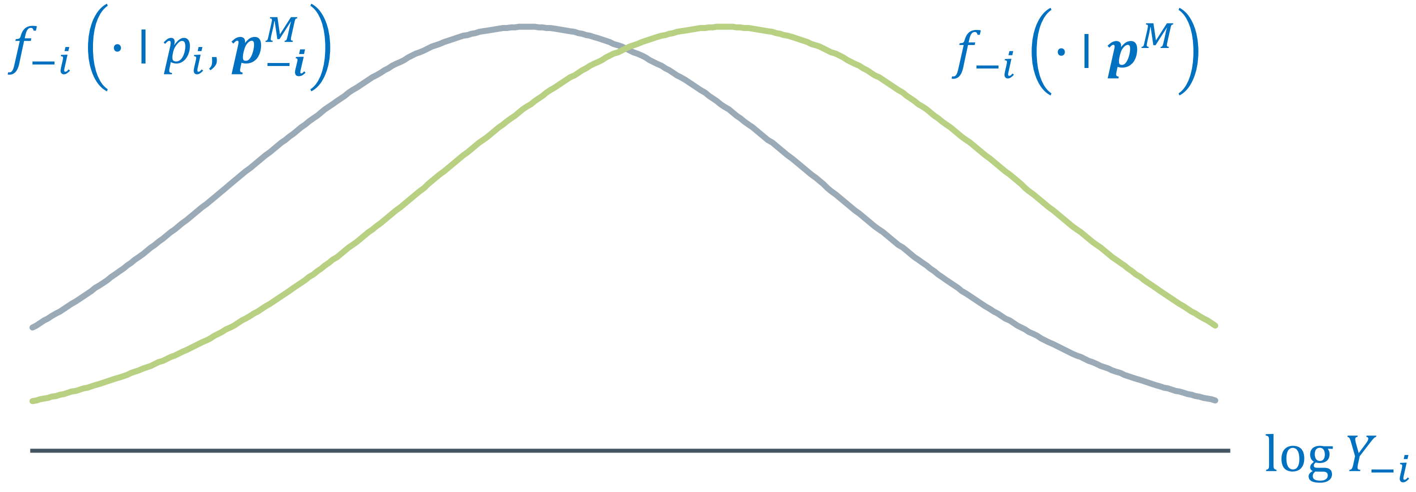
› Price elasticity varies period by period

Good monitoring



small $\sigma^2 \Rightarrow$ large η

Poor monitoring



large $\sigma^2 \Rightarrow$ small η

Monitoring quality

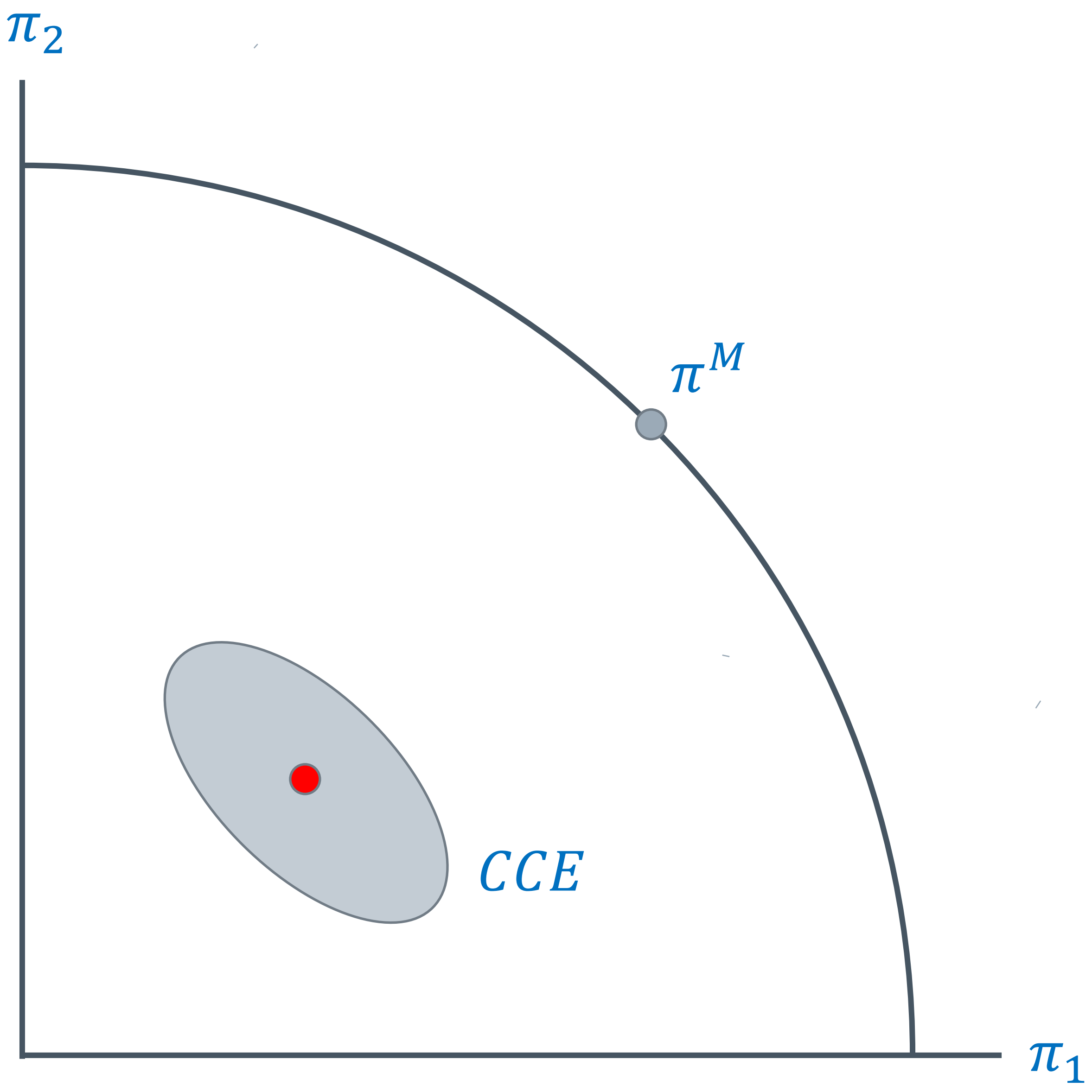
- With (log) normal sales

$$\eta \propto \frac{1}{\sigma^2}$$

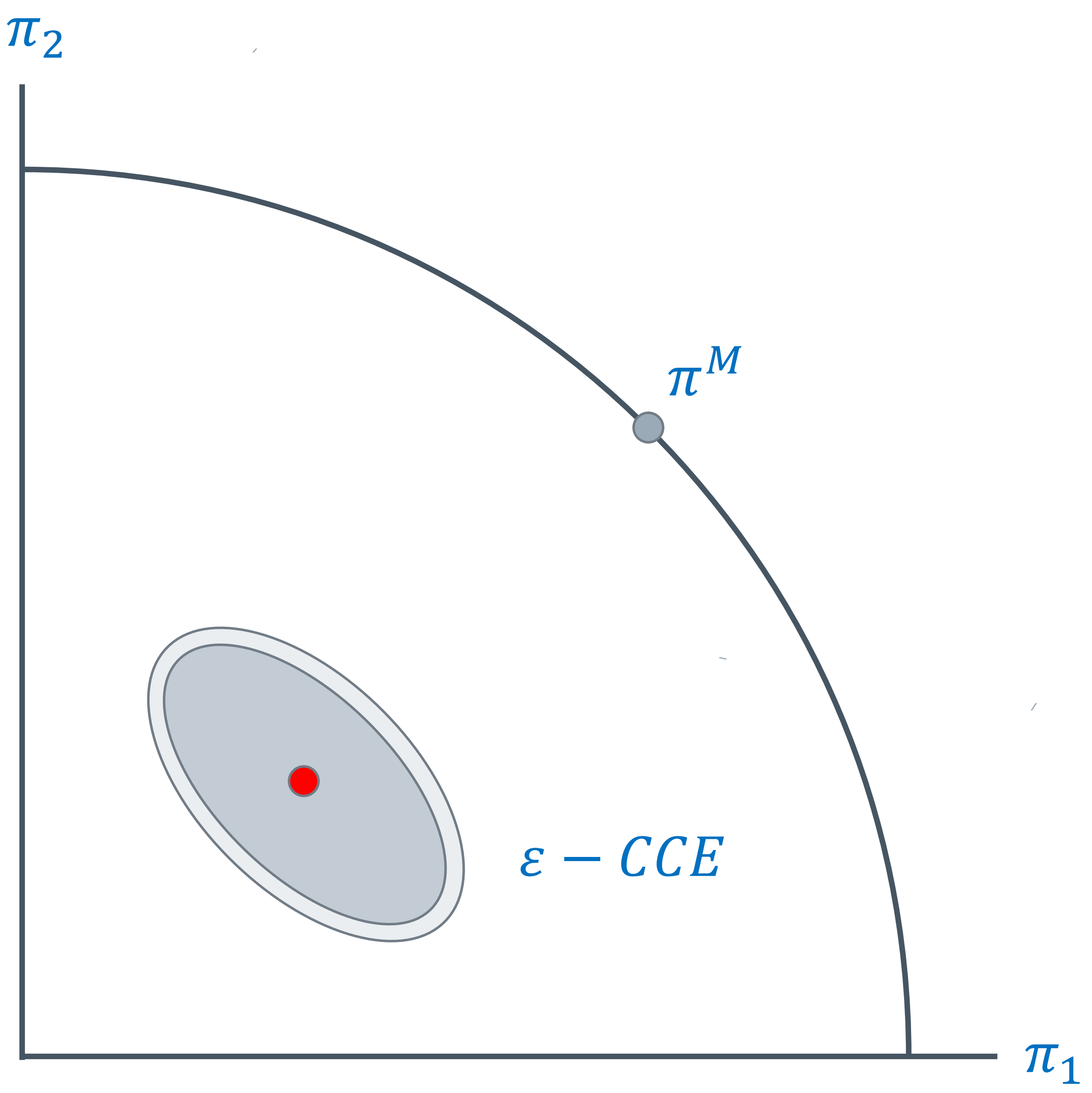
Tacit vs. Explicit Collusion

Theorem: Fix $\delta > \underline{\delta}$.

When σ^2 and $\rho(\boldsymbol{p}^M)$ are large, there is an eqm w/ comm whose profits exceed those in any eqm w/o comm.



π2
πM
CCE
π1


π_2
π^M
$\varepsilon - CCE$
π_1

Application 2: Indirect Communicatin

› Antitrust law allows exchange of aggregate information

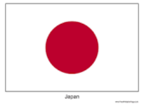
Policy

- European Commission (2011)
  - “Exchanges of genuinely aggregated data, that is to say, where the recognition of individualised company level information is difficult, are much less likely to lead to restrictive effects on competition than exchanges of company level data.”

Policy

- US Federal Trade Commission (2014)
  - [Companies should ensure that] "the shared statistics are sufficiently aggregated that no participant can discern the data of any other participant."

Policy

- 

Japan Fair Trade Commission (2015)
  - [The exchange of information] "without clearly indicating the quantities, amounts etc. of individual constituent enterprises" [is permissible.]

Policy

- Carlton, Gertner and Rosenfield (1997)
  - "[…] aggregating the data largely removes the value of information in facilitating collusion."

Theory of repeated games

- RMM (1986) example: With only aggregate information, full cooperation is not possible even with patient players.
- FLM (1994) folk theorem: With pairwise identifiability, cooperation can be sustained with patient players

Policy & theory

Policy based on the idea, supported by theory, that

› Detection of *cheating* is not sufficient for cooperation

› Detection of *cheater* is needed

Cartels

- Amino acids
  - "ADM stated that the way for them to communicate is through a trade association. ADM explained by way of example that ADM reported its citric acid sales every month to a trade association, and every year Swiss accountants audited those figures." (EC 2001)
- Copper tubes
- Zinc phosphate
- …
- 11 out of 22 cartels used third parties (Marshall & Marx 2012)

Model

- Expected demand $\mathbb{E}[\,Y_i \mid \boldsymbol{p}]$ $\downarrow$ in p_i and $\uparrow$ in p_j
- Expected profits $\pi_i = p_i\,\mathbb{E}[\,Y_i \mid \boldsymbol{p}]$

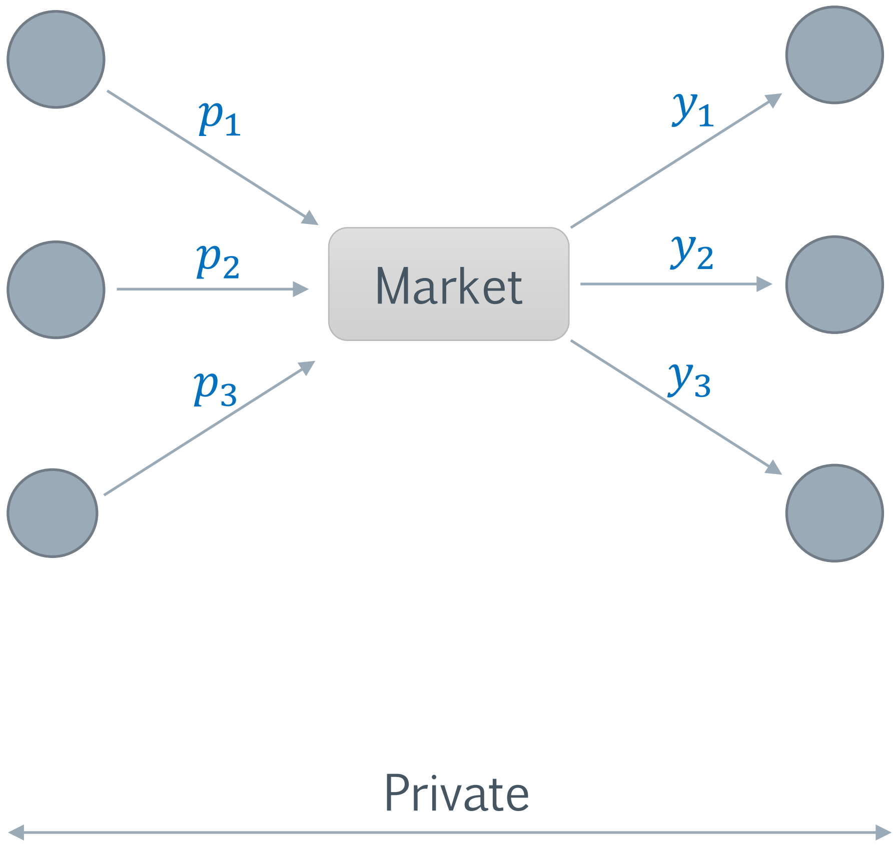
Model

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \cdots & \rho \\ \vdots & \ddots & \vdots \\ \rho & \cdots & 1 \end{bmatrix}$$

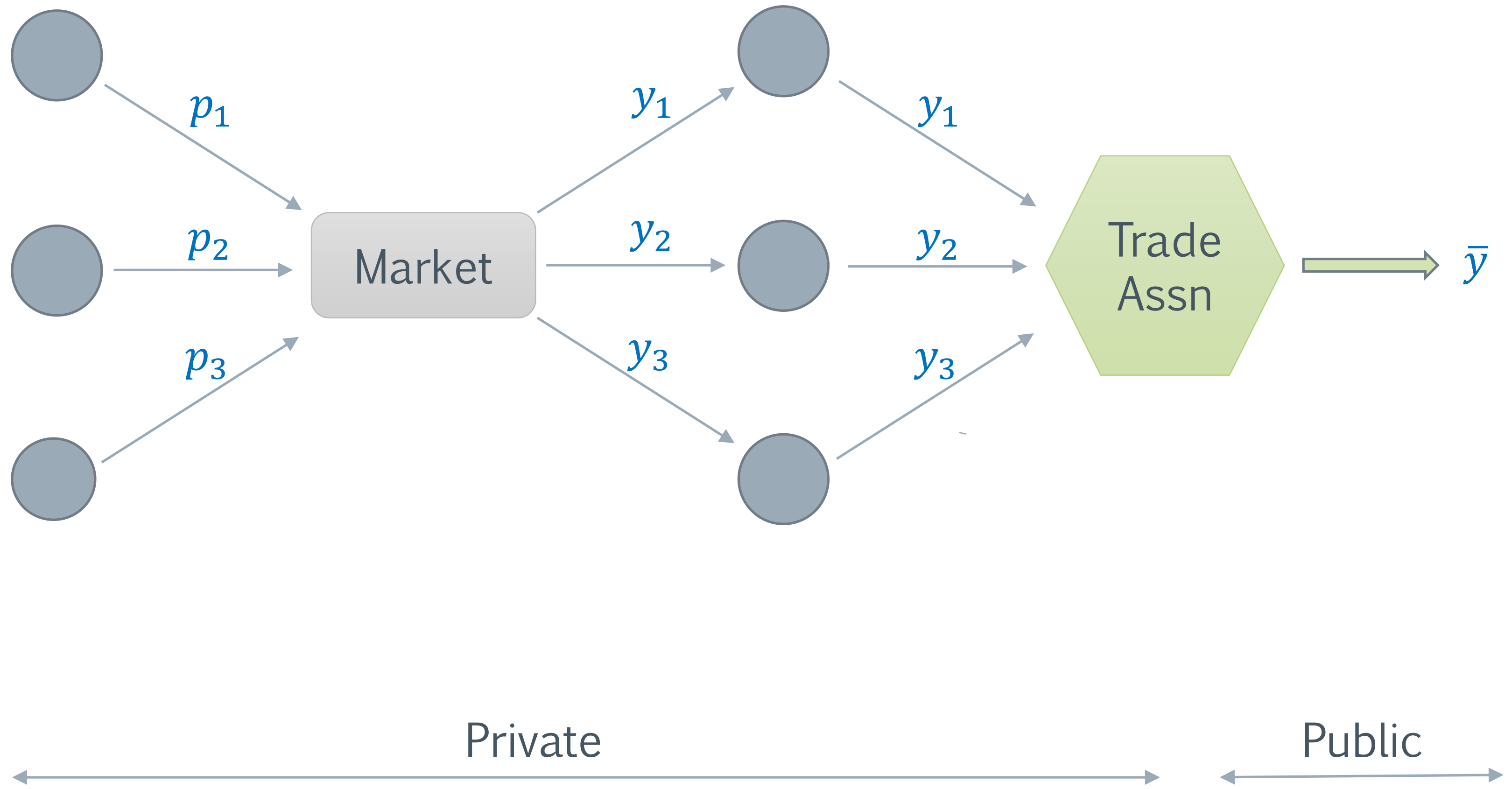
$$-\frac{1}{n-1} < \rho < 0$$

› Note that prices affect only expected sales.

Scenario 1



Scenario 2



p_1
p_2
p_3
Market
y_1
y_2
y_3
y_1
y_2
y_3
Trade Assn
$\bar{y}$
Private
Public

Main result

Proposition: Fix $\delta > \underline{\delta}$. There exist $\underline{\sigma} > 0$ and $\overline{\rho} < 0$ such that for all $\sigma > \underline{\sigma}$ and $\rho < \overline{\rho}$ there is an equilibrium with aggregate sales in which total profits are strictly larger than in any equilibrium without.

Summary

- A new method for analyzing repeated games with private monitoring
  - Bound on set of equilibrium payoffs for arbitrary discounting

- Application to tacit vs. explicit collusion question
- Application to collusive effects of exchanging aggregate information