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The art of conversation: eliciting information from experts through multi-stage communication ☆

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Abstract

We examine the strategic interaction between an informed expert and an uninformed decision maker, extending the analysis of Crawford and Sobel (Econometrica 50 (1982) 1431). We modify their model to allow for more extensive communication between the two parties and show that face-to-face communication between the expert and the *uninformed* decision maker followed by a written report from the expert leads to improved information transmission. In (almost) all cases, there exists an equilibrium in our modified model that ex ante Pareto dominates all of the equilibria identified by Crawford and Sobel. This remains true even if the expert's bias is so great that in their model no information would be disclosed.

JEL classification: C72; C82

Keywords: Communication; Information; Experts; Cheap talk

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1. Introduction

In many situations of economic interest, those with the power to make decisions lack important information about the economic consequences of their choices. As a result, decision makers often seek advice from better informed parties—experts—prior to making decisions. Examples of such situations abound. CEOs consult investment bankers, strategic planners, and marketing specialists before making corporate decisions. Congressional representatives hold hearings and consult lobbyists to learn more about the impact of proposed legislation. Investors read reports of equity analysts and call up stock brokers for advice and tips before deciding on an investment strategy.

A feature common to all of these situations is that the expert being consulted may well have preferences that do not coincide with those of the decision maker. As a result, the expert may have the incentive to mislead or to withhold information. In such situations it is important that the decision maker be able to elicit as much information as possible from the expert. Indeed, the ability to do this is commonly thought of as the mark of an effective leader.

The strategic interaction between an uninformed decision maker and an informed expert was first studied by Crawford and Sobel [7] (hereafter 'CS') in a now classic paper. In their model the expert, after learning the realization of the payoff relevant state of nature, sends a costless message to the decision maker, who then takes an action that has consequences for both parties. Interest in the problem arises, of course, from the assumption that the preferences of the two parties are not perfectly aligned. Crawford and Sobel obtain a complete characterization of the set of equilibria in their model and identify the Pareto dominant equilibrium. They show that preference divergence between the two parties inevitably leads to withholding of information by the expert; that is, full revelation is never an equilibrium outcome. Further, as the degree of preference divergence increases, the amount of information disclosed by the expert decreases. Once the preference divergence is sufficiently large, the expert can credibly disclose no information whatsoever.

There are two important features regarding the structure of communication in their model. First, the role of the decision maker in eliciting information from the expert is completely passive. Second, there is only a single stage of communication between the two parties before the decision is made. A number of decision making situations are like this. For instance, sell-side equity analysts send reports detailing their recommendations regarding a particular stock to investors who may then use this information to make an investment decision. Individual investors do not consult analysts during report preparation and decisions are taken after one stage of communication.

In other situations, however, the structure of communication is more extensive and may entail active participation by the decision maker and multiple stages of communication. For instance, CEOs employing strategy consultants often hold a series of face-to-face meetings where they offer input leading up to a report or formal presentation. One explanation immediately suggested for differences between the structure of communications in this situation as compared to the investor seeking advice from an analyst is that the CEO himself might possess useful information apart from that of the expert. In this case, it seems plausible that active dialog between the decision maker and the expert might be beneficial and that multiple stages might be needed so that the expert has a chance to incorporate the new information received from the CEO into his final recommendation.

In the extreme situation considered in the CS model—the decision maker has no information not already held by the expert—the usefulness of this structure of communication would seem dubious. The main lesson of this paper is that, even when the decision maker himself possesses no useful information, his active participation combined with multiple stages of communication leads to greater information disclosure by the expert and this redounds to the benefit of *both* parties in terms of payoffs.

In particular, we show how a simple and quite natural modification of the mode of communication in the CS model can result in improvements in information transmission. In the first stage, the informed expert and the uninformed decision maker engage in one round of "conversation," which we model simply as a simultaneous exchange of cheap talk signals. After this, in a second stage, the expert may send a further message, say a written report. The key aspect of our modified model is that the communication takes place over multiple stages and that, in the first stage, it involves two-sided communication—that is, the uninformed decision maker is an active participant in the process. At first, it seems incredible that communication by a party that has nothing to communicate can have any effect. Indeed, since the decision maker is completely uninformed his communication can only consist of random messages. We show, however, that the introduction of random elements can lead to more information being conveyed and higher payoffs to both parties despite (as we show later, because of) the fact that both parties are risk-averse.

The main results in our paper establish that in (almost) all circumstances where the decision maker can extract some information from the expert in the CS model, there exists an equilibrium in our modified model that ex ante Pareto dominates all of the equilibria identified by Crawford and Sobel (Theorem 1). Further, even when the expert's bias is sufficiently great that the decision maker can obtain no information in the CS model, he might can still obtain information in our modified model as long as the preference divergence is not too extreme (Theorem 2).

Where do these payoff gains come from? We show that, when preference divergence is not too large, *risk-aversion* plays an important role in generating payoff gains to both parties from conversation. In the situation considered in the CS model, increases in risk-aversion make payoff gains from an additional round of conversation more likely. Since the purpose of injecting the active participation of the decision maker into the conversation was purely to introduce randomness, it is surprising that introducing randomness improves payoffs only when the parties are sufficiently risk-averse. Later in the paper, we offer a partial explanation for this seeming paradox.

1.1. Relation to the literature

It is known that adding rounds of communication can expand the sat of equilibrium outcomes even in games with complete information. In two player games, however, these effects are rather limited. In such games, the set of equilibrium payoffs with preplay communication is just the convex hull of equilibrium payoffs of the original game (see, for instance, [3,12]).

In a double auction with incomplete information, Farrell and Gibbons [9] and Matthews and Postlewaite [18] show that additional equilibria arise when that game is modified by adding a single round of simultaneous cheap talk between a buyer and a seller. Also along these lines is the paper by Forges [11]. She constructs an extended example concerning an uninformed employer and a job applicant with private information about his type. With only a single round of communication, in her example, there is a separating and a pooling equilibrium. She shows how adding multiple rounds of costless signaling affects the set of equilibrium payoffs. While multiple rounds permit payoffs that are better for the applicant this is not so for the employer since the employer can do no better than in the fully separating equilibrium of the game with one stage of communication.

Aumann and Hart [1] provide a complete characterization of the set of Bayes–Nash equilibria in two-person bimatrix games where one of the players is better informed than the other and where conversation consists of up to an infinite number of stages of communication. Their equilibrium characterization is geometric in nature, using the newly introduced concepts of *diconvexity* and *dimartingale*. It is quite general in scope—it applies to all bimatrix games, not just "cheap talk" games.[1] The cost of this generality, however, is that the characterization is quite abstract. As a consequence, even in special classes of games, it is not understood when additional communication leads to Pareto improvements nor why.

Our contribution relative to this literature is to show that in the CS model the introduction of an additional round of communication does not simply expand the set of equilibria, but expands it in such a way that the equilibrium payoffs of both the decision maker and the expert can (almost) *always* be improved—to the mutual benefit of both the decision maker and the expert. We show that in the CS model, the key to this result is that the informed party be sufficiently risk-averse. To our knowledge, we are the first to demonstrate the possibility of *Pareto* gains from adding rounds of signaling beyond the first and that risk-aversion is responsible for these payoff improvements.

These results are not simply of theoretical interest. The CS model constitutes a foundation for work on costless communication. It has been studied extensively and applied to problems in political science (see the recent book by Grossman and Helpman [15] for an account of this work), public finance [6], finance [19], and other fields. The CS model is the benchmark used to compare the benefits of introducing multiple experts [5,16], delegation [8], screening [4], and restrictive legislative rules

[1] Since the Aumann and Hart characterization concerns games with finite states and actions, it is not possible to apply their results to a setting—like the CS model—with a continuum of states and actions.

[13,14,17]. The pessimistic estimates of information loss contained in Crawford and Sobel may thus affect policy conclusions based on these comparisons.

Finally, a separate strand of the literature deals with the case where an external *mediator* may be used to convey information between the parties (see [10,12,20]). In contrast, we are interested in the effects of *plain conversation*, where no mediation is possible. In Section 6 we demonstrate that this distinction matters. In an example we construct an equilibrium when the expert is free to use a mediator that is not attainable with plain conversation.

The remainder of the paper proceeds as follows: In Section 2, we review the CS model and highlight the central properties of their equilibrium characterization. Section 3 presents two examples which illustrate our main result—the introduction of an additional round of communication between the expert and the decision maker leads to improved information transmission. In Section 4, we prove this result and characterize a class of monotonic equilibria for the case where the expert's bias is not too large. Section 5 deals with the extreme bias case and shows that nonmonotonic equilibria are required to improve information transmission in these circumstances. Section 6 examines some extensions, especially the effect of mediated talk. Section 7 discusses the results.

2. The Crawford–Sobel model

We consider the uniform-quadratic model introduced by Crawford and Sobel [7].[2] A decision maker must choose some action y. His payoff from this decision depends on the action and an unknown the state of the world, θ, assumed to be distributed uniformly on the unit interval.

The decision maker can base his decision on the message, m, sent at no cost by an expert who knows the precise value of θ. The decision maker's payoff function is

$$U(y,\theta) = -(y-\theta)^2 \tag{1}$$

and the expert's payoff function is

$$V(y,\theta,b) = -(y-(\theta+b))^2, \tag{2}$$

where $b > 0$ is a parameter that measures how closely the preferences of the expert are aligned with those of the decision maker. In other words, b is a measure of the *bias* of the expert relative to that of the decision maker.

Note that if the preferences of the two parties are identical—that is, if $b = 0$, then it is in the interests of the expert to reveal the state precisely.

[2] While the equilibrium characterization obtained by Crawford and Sobel applies to a more general specification of preferences and state distributions, to make welfare comparisons among the equilibria requires much more structure (their Assumption M). In practice, the uniform-quadratic model is the only specification that is used in applications of their model. In Section 4.3 we consider a more general family of preferences.

Crawford and Sobel postulate the following sequence of play:

Expert learns θ	Expert sends written message m	DM choose action y

The communication is one-sided—from the informed expert to the uninformed decision maker—and comes in a single stage. In what follows, we refer to such one-sided communication as being in *written* form. This serves to contrast this form of communication with the form studied in later sections.

Crawford and Sobel have shown that all equilibria in their game are equivalent to *partition equilibria*, that is, equilibria in which there is only a finite number of actions chosen in equilibrium and each action is associated with an interval of states. This also implies that the equilibria are *monotonic*—that is, the equilibrium action is a nondecreasing function of the state.

Crawford and Sobel also show that for any value of b there are a *finite* number of equilibrium outcomes.

Let

$$N(b) = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2}{b}} \right\rceil, \tag{3}$$

where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. Crawford and Sobel show that the number of distinct equilibrium outcomes is finite—it is exactly $N(b)$. There is exactly one equilibrium with a one-element partition, one equilibrium with a two-element partition, ... , one equilibrium with $N(b)$-element partition. There is no equilibrium in which the number of elements of the partition exceeds $N(b)$. Moreover, the $N(b)$ equilibria can be Pareto ranked and both parties prefer equilibria with a greater number of partition elements. That is, for any $b > 0$, the number of elements of the partition associated with the *Pareto dominant* equilibrium, which we will term the *best* equilibrium, is $N(b)$.

Define

$$\beta(N) = \frac{1}{2N(N+1)}. \tag{4}$$

It is routine to verify that for all b such that $\beta(N) \leqslant b < \beta(N-1)$, $N(b) = N$. Crawford and Sobel show that, for all b such that $\beta(N) \leqslant b < \beta(N-1)$, the best equilibrium is one in which the state space is partitioned into N intervals $[a_{i-1}, a_i]$, $i = 1, 2, \dots, N$, where

$$a_j = \frac{j}{N} + 2bj(j - N). \tag{5}$$

In such an equilibrium, the expert sends the same message, m_i, if the state $\theta \in [a_{i-1}, a_i]$ and, given this message, the decision maker takes the optimal action

$$y_i^* = \frac{a_{i-1} + a_i}{2}.$$

The expected payoff of the decision maker in the *best* equilibrium—that is, one with an $N(b)$ element partition—is

$$U(b) = -\frac{1}{12N(b)^2} - \frac{b^2(N(b)^2 - 1)}{3}. \tag{6}$$

Finally, note that if the bias of the expert is large enough, then the expert will convey no information to the decision maker. Specifically, if $b \geqslant \frac{1}{4} = \beta(1)$, then $N(b) = 1$ and so the expert sends the same message for all $\theta \in [0, 1]$.

3. Face-to-face communication

Our goal is to explore how amendments to the original model of Crawford and Sobel that allow for more extensive communication, possibly involving both the parties, affect information transmission. Our main results show that active participation by the decision maker along with multiple stages of communication lead to more information disclosure by the expert. Before proceeding with the analysis of this model, it is useful to show that either feature on its own does not lead to an improvement.

As a first step, consider a variation of the CS model in which there are multiple stages but where the decision maker is passive. That is, the expert sends multiple messages, say l and m, to the decision maker as depicted below:

Expert learns θ	Expert sends message l	Expert sends message m	DM chooses action y

This, however, does not affect the set of equilibria identified by Crawford and Sobel. This is because the expert can anticipate the actions $y(l, m)$ that the decision maker will choose following any pair of messages l and m, and so the expert will choose a pair of messages that maximizes his expected utility in state θ. But these outcomes are the same as CS equilibria. More generally, if there is a deterministic rule that determines the decision maker's choices as a function of the messages sent by the expert, the set of equilibria are the same as in the CS model. Thus, multiple stages per se are not beneficial.

Next, consider a variation of the CS model where the decision maker is an active participant but there is a single stage. That is, consider the sequence of play where:

Expert learns θ	Expert and DM each send a message	DM chooses action y

In this case also, regardless of whether the messages are exchanged sequentially or simultaneously, the set of equilibria is identical to the set of CS equilibria. This is obvious when messages are exchanged simultaneously or when the decision maker sends his message after the expert's since the decision maker has no useful information of his own. When the decision maker sends his message first, this message could, in principle, be used as a randomizing device to coordinate among the set of possible equilibria in the CS model. The decision maker, however, is not indifferent among any of the equilibria and so would not want to randomize among these. Thus active participation by the decision maker per se is not beneficial either.

We study the simplest possible model which combines both features—active participation by the decision maker and multiple stages of communication—and show that together they lead to benefits. Specifically, we amend the original model of Crawford and Sobel to allow the expert and the decision maker to meet face-to-face in the first stage and engage in (simultaneous) communication. In the second stage, the expert is allowed to send a further message, possibly conditioning this on the outcome of the face-to-face meeting. In the amended model the sequence of play is:

Expert learns θ	Expert and DM meet face-to-face	Expert sends written message m	DM chooses action y

A word on terminology is in order here. We use the term "face-to-face" meeting as short-hand to define a situation in which the two parties send messages l_1 and l_2 simultaneously. (This should be contrasted with sequential communication in which the parties speak sequentially, that is, first player 1 sends the message l_1 and then 2, having heard l_1 sends the message l_2.) It is useful to think of this situation as a meeting between the expert and the decision maker prior to the issuance of a written report by the expert. In what follows, we will refer to the two-stage communication structure outlined above as the *model with face-to-face communication*.

We begin by demonstrating the benefits of this mode of communication in the context of two specific examples. Example 1, illustrating the workings of one of our main results, Theorem 1, shows how face-to-face communication can improve information when the expert is "moderately" biased. In Example 2, illustrating Theorem 2, the expert's bias is extreme—that is, with only written communication, the only equilibrium involves babbling and so *no* information is revealed. The example illustrates that even in this case, face-to-face communication can lead to useful information transmission.

Example 1. Suppose that the expert's bias $b = \frac{1}{10}$. Then there are exactly two equilibria in the CS model.[3] One is the completely noninformative (or "babbling") equilibrium in which the expert sends the same message regardless of the state, and the decision maker disregards all messages from the expert. The other equilibrium is partially informative, and leads to a partition in which the expert breaks $[0, 1]$ at $a_1 = \frac{3}{10}$; that is, the only information that the expert reveals is whether the state θ lies in the interval $[0, \frac{3}{10}]$ or in the interval $[\frac{3}{10}, 1]$. If $\theta \in [0, \frac{3}{10}]$, the decision maker takes the appropriate optimal action $y = \frac{3}{20}$ and if $\theta \in [\frac{3}{10}, 1]$, the decision maker takes the optimal action $y = \frac{13}{20}$. The ex ante expected payoff of the decision maker in this equilibrium is $-\frac{37}{1200}$ whereas the ex ante expected payoff of the expert is $-\frac{49}{1200}$.

[3] Technically, there are a continuum of equilibria all of which are outcome equivalent to one of the two described in the text.

Let us now amend the model to allow for face-to-face communication between the informed expert and the uninformed decision maker. The following strategies constitute an equilibrium of the extended game.

The face-to-face meeting consists of a simultaneous exchange of messages between the expert and the decision maker. The expert reveals some information at the meeting but there is also some randomness in what transpires. Depending on how the conversation goes, the meeting is deemed by both parties to be a "success" or it is deemed to be a "failure." How this is done is explained in more detail below.

During the meeting, the expert reveals whether the state, θ, is above or below $x = \frac{2}{10}$; he also sends some additional messages that affect the success or failure of the meeting.

If he reveals during the meeting that $\theta \leqslant \frac{2}{10}$, then any other messages exchanged in the meeting, and in the subsequent written report, are uninformative. The decision maker then plays a low action $y_L = \frac{1}{10}$ that is optimal given the information that $\theta \leqslant \frac{2}{10}$.

But if the expert reveals during the meeting $\theta > \frac{2}{10}$, then the informativeness of the written report depends on whether the meeting was deemed to be a success or a failure. In the event of a failure, there is no further information contained in the subsequent written report. The decision maker then plays the "pooling" action $y_P = \frac{6}{10}$ that is optimal given the information that $\theta > \frac{2}{10}$. In the event the meeting is a success, however, information in the written report results in a further division of the interval $[\frac{2}{10}, 1]$ into $[\frac{2}{10}, \frac{4}{10}]$ and $[\frac{4}{10}, 1]$. In the first subinterval, the medium action $y_M = \frac{3}{10}$ is taken and in the second subinterval the high action $y_H = \frac{7}{10}$ is taken. The actions taken in different states are depicted in Fig. 1. The dotted line depicts the action, $\theta + \frac{1}{10}$, that is "ideal" for the expert in each state.

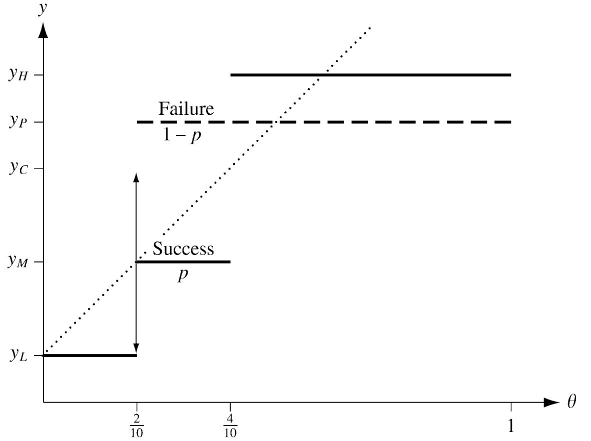

Fig. 1. Equilibrium with face-to-face meeting.

Table 1
A jointly controlled lottery

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
A_1	1	0	0	0	0	1	1	1	1
A_2	1	1	0	0	0	0	1	1	1
A_3	1	1	1	0	0	0	0	1	1
A_4	1	1	1	1	0	0	0	0	1
A_5	1	1	1	1	1	0	0	0	0
A_6	0	1	1	1	1	1	0	0	0
A_7	0	0	1	1	1	1	1	0	0
A_8	0	0	0	1	1	1	1	1	0
A_9	0	0	0	0	1	1	1	1	1

Notice that in state $\frac{2}{10}$, the expert prefers y_L to y_P (in the figure, y_L is closer to the expert's ideal point than is y_P) and prefers y_M to y_L (indeed y_M is the expert's ideal action).[4] Thus, if there were no uncertainty about the outcome of the meeting—for instance, if all meetings were "successes," then the expert would not be willing to reveal whether the state is above or below $\frac{2}{10}$. In particular, for states $\theta = \frac{2}{10} - \varepsilon$, it would be in the interests of the expert to say that $\theta > \frac{2}{10}$ thereby inducing y_M instead of y_L. Similarly, if all meetings were failures, then for states $\theta = \frac{2}{10} + \varepsilon$, it would be in the interests of the expert to say $\theta < \frac{2}{10}$, thereby inducing y_L instead of y_P.

Clearly, there exists a probability p such that in state $\frac{2}{10}$ the expert is indifferent between y_L and a $p:1-p$ lottery between y_M and y_P (whose certainty equivalent is the action labelled y_C in the figure). It is also the case that for all $\theta < \frac{2}{10}$, the expert prefers y_L to a $p:1-p$ lottery between y_M and y_P; for all $\theta > \frac{2}{10}$, the expert prefers a $p:1-p$ lottery between y_M and y_P to y_L. It may be verified that in this example, $p = \frac{5}{9}$.

It remains to specify a conversation in the meeting such that it is deemed to be a success with probability $p = \frac{5}{9}$. To do this we employ the device of constructing a *jointly controlled lottery* (see [2]). Let the expert send a message of the form (Low, A_i) or $(High, A_i)$ and let the decision maker send a message of the form A_j, where $i, j \in \{1, 2, \ldots, 9\}$. These messages are interpreted in the following manner. *Low* signals that $\theta \leqslant \frac{2}{10}$ and *High* signals that $\theta > \frac{2}{10}$. Thus the first component of the expert's message conveys some (coarse) information. The A_i and A_j messages play the role of a coordinating device and determine whether the meeting is deemed to be a success or not. The expert chooses A_i at random, and each A_i is equally likely. Similarly, the decision maker chooses A_j at random. Given these choices, the meeting is deemed to be a success if $0 \leqslant i - j < 5$ or if $j - i > 4$. Otherwise, it is a failure. (The possible outcomes are provided in Table 1 in which a 1 denotes a success and a 0 denotes a failure.) For example, if the messages of the expert and the decision maker

[4] The fact that y_M is the ideal point for the expert in state x is a artifact of the example and unimportant for this construction. The general principle is that the expert prefer y_M to y_L.

are $(High, A_2)$ and A_8, respectively, then it is inferred that $\theta > \frac{2}{10}$ and since $j - i = 6 > 4$, the meeting is declared to be a success. Observe that with these strategies, given any A_i, the probability that the meeting is a success is exactly $\frac{5}{9}$. Similarly, given any A_j, the probability of success is $\frac{5}{9}$ also.

It may be verified that this indeed constitutes an equilibrium. In particular, given the proposed play in the subgames, in every state $\theta < \frac{2}{10}$ the expert prefers to send the message *Low*, and in every state $\theta > \frac{2}{10}$ the expert prefers to send the message *High*. Moreover, given the strategies neither the expert nor the decision maker can affect the play in the subgame by strategically sending the various coordinating messages A_i or A_j.

The equilibrium of the extended game constructed above conveys more information to the decision maker than any of the equilibria of the CS model. In face, it is ex ante Pareto superior. The ex ante expected payoff of the decision maker is now $\frac{36}{37} \times \left(-\frac{37}{1200}\right)$ whereas that of the expert is $\frac{48}{49} \times \left(-\frac{49}{1200}\right)$. The remarkable fact about the example is that this improvement in information transmission is achieved by adding a stage in which the *uninformed* decision maker also participates by injecting uncertainty into the resulting actions despite the fact that both parties are risk-averse.

Both parties coordinate their play in the subsequent game on the outcome of the meeting. Since the outcome of the meeting is uncertain—whether it is a success or failure is not known beforehand—this creates uncertainty over how the information conveyed by the expert will ultimately translate into decisions. This in turn, alters the incentives for the expert to reveal divisions of the state space (in the original CS model, having the expert report that θ is above or below $\frac{2}{10}$ cannot be part of any equilibrium). The informational gains result from the fact that, having already revealed a division of the state space in the meeting preceding the written report, it is now credible for the expert to convey more precise information in his report. Although this happens only probabilistically, it is beneficial ex ante.

Example 2. We now turn to situations where the bias of the expert is extreme. In particular, suppose that the expert is so biased $(b \geqslant \frac{1}{4})$ that no information transmission takes place in the CS model—that is, the only equilibrium involves babbling. Can a quiescent expert be induced to reveal some information as a result of a face-to-face meeting? Surprisingly, the answer turns out to be yes.

Specifically, suppose $b = \frac{7}{24}$. Let $x = 0.048$ and $z = 0.968$ (the reasons for particular choices of these points will become apparent later). Suppose that during the face-to-face meeting the expert reveals whether the state θ is in the set $[x, z]$ or not. If he reveals that $\theta \in [x, z]$, then the meeting ends and there is no further information contained in the written report. The decision maker then plays a medium action $y_M = \frac{1}{2}(x + z)$ that is optimal given the information that $\theta \in [x, z]$.

If the expert says $\theta \in [0, x] \cup [z, 1]$, then, as in Example 1, the informativeness of the written report depends on whether the meeting was deemed to be success or a failure. In the event of a failure, once again the written report contains no further information, and the decision maker plays the pooling action, $y_P = 0.407$, that is optimal given the information that $\theta \in [0, x] \cup [z, 1]$. In the event the meeting was a success, information contained in the written report results in a further division of the set $[0, x] \cup [z, 1]$ into $[0, x]$ and $[z, 1]$. In the first subinterval, the low action $y_L = \frac{x}{2}$ is taken and in the second subinterval the high action $y_H = \frac{1+z}{2}$ is taken. See Fig. 2.

In state x, the expert prefers y_M to y_L but prefers y_P to y_M. In state z, however, the expert prefers y_H to y_M but prefers y_M to y_P. It can be shown that there exists a probability p such that the expert is indifferent between y_M and a $p : 1 - p$ lottery between y_L and y_P when the state is x. At the same time, the expert is indifferent between y_M and a $p : 1 - p$ lottery between y_H and y_P when the state is z. In this example, this is true for $p = \frac{1}{4}$.

A conversation such that the meeting is deemed to be a success with probability $p = \frac{1}{4}$ is easily specified as before. Let the expert send a message of the form (In, A_i) or (Out, A_i) and let the decision maker send a message of the form A_j, where $i, j \in \{1, 2, 3, 4\}$. These messages are interpreted in the following manner. The signal *In* means that $\theta \in [x, z]$ and *Out* means that $\theta \in [0, x] \cup [z, 1]$. The messages A_i and A_j are chosen randomly with equal probability and the meeting is deemed a success if and only if $i = j$.

Under the nonmonotonic equilibrium the expected payoffs to both the decision maker and the expert are higher than under the babbling equilibrium. The decision maker earns an expected payoff of -0.078 under the nonmonotonic equilibrium as opposed to -0.083 under babbling. Likewise, the expert earns an expected payoff of -0.163 under the nonmonotonic equilibrium as compared to -0.168 under babbling. The key point is that since there is some information being transmitted in this equilibrium, the outcome is Pareto superior to the unique (babbling) equilibrium of the CS model. Thus a face-to-face meeting preceding a written report

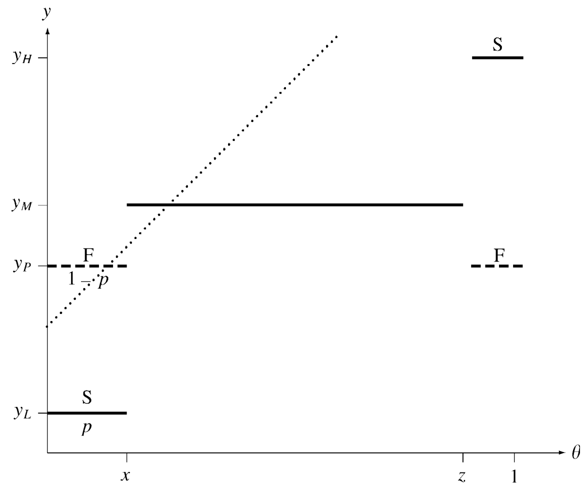

Fig. 2. A nonmonotonic equilibrium.

can result in information transmission in circumstances where a written report by itself would be completely uninformative.

As in the previous example, the introduction of uncertainty in final decisions relaxes the incentives of the expert to reveal truthfully. While that remains important, the key to this construction is that the initial information conveyed by the expert is "non-convex": in some cases he reveals only that $\theta \in [0, x] \cup [z, 1]$. Then, if the meeting is a success, the expert is willing to further reveal whether $\theta \in [0, x]$ or $\theta \in [z, 1]$. If the initial information were an interval, as in Example 1, the extreme bias of the expert would preclude any further information disclosure. (In fact, we show in Proposition 3 that once the bias is sufficiently high $(b \geqslant \frac{1}{8})$, there is no monotonic equilibrium of the game with a face-to-face meeting that is superior to all equilibria of the CS model.)

The key to both examples is that at after the expert reveals some information in the face-to-face meeting, there are multiple equilibria in the remaining game. In Example 1, once the expert reveals that the state is in $[x, 1]$, there are two equilibria—the babbling equilibrium and a partition equilibrium of size 2. In Example 2, the expert's bias is so high that there are no nontrivial partition equilibria and so this trick does not work. But if the expert instead reveals that the state is in a set of the form $[0, x] \cup [z, 1]$, then in the remaining game there are two equilibria—the babbling equilibrium and on in which this set is further subdivided into $[0, x]$ and $[z, 1]$. The role of the decision maker is to ensure that which of the two equilibria is played depends on the outcome of a random event—whether or not the conversation is successful—and the probability of this is chosen so that the expert is willing to reveal the relevant information during the face-to-face meeting.

4. Moderate bias $(b < \frac{1}{8})$

In this section we generalize the construction in Example 1 to show that, when an expert's bias is moderate $(b < \frac{1}{8})$, there exist equilibria with conversations that are Pareto superior to all equilibria in the CS model. The main result in this section is:

Theorem 1. *For almost all $b < \frac{1}{8}$, there exists a perfect Bayesian equilibrium in the model with face-to-face communication which is Pareto superior to all equilibria in the CS model.*

To prove the result, we first characterize a class of equilibria arising with face-to-face conversations (Section 4.1) and then show that for almost all values of b, the payoff maximizing equilibrium in this class is an improvement over all equilibria in the CS model (Section 4.2). In Section 4.3, we consider a more general class of preferences and offer some intuition for the welfare results highlighting the role played by the expert's risk-aversion.

For future reference, it is useful to note that, like all partition equilibria, the equilibria constructed in this section are all *monotonic*—that is, they have the property that higher states are always associated with (stochastically) higher actions.[5] The main result of this section pertains to situations in which the bias $b < \frac{1}{8}$.

4.1. Construction of equilibria

The nature of the equilibrium, whose detailed construction is provided below, is as follows.

- In the face-to-face meeting, the expert's message reveals whether the state θ is less than a given quantity x or not and a second message A_1 chosen from some suitable set of messages $\mathscr{A}$. Formally, the expert's message is either of the form (Low, A_1) or of the form $(High, A_1)$. The first component, *Low* or *High*, conveys whether $\theta < x$ or not. The decision maker also sends a message $A_2 \in \mathscr{A}$.
- Subsequent play depends on the messages that were exchanged in the face-to-face meeting.
  - If the expert says *Low*, this is interpreted to mean that $\theta < x$. In the remaining game, a partition equilibrium in the interval $[0, x]$ is played regardless of what other messages are exchanged.
  - If the expert says *High*, this is interpreted to mean that $\theta \geqslant x$. The subsequent play then depends on whether the meeting is deemed to be a success. The success or failure of the meeting is determined by a "success function" $S : \mathscr{A} \times \mathscr{A} \to \{0, 1\}$. A meeting is a success if and only if $S(A_1, A_2) = 1$. The message sets $\mathscr{A}$ and the success function S are chosen so that a particular jointly controlled lottery to be chosen below can be played.
    - If the meeting is deemed to be a success, then in the subsequent play the expert further reveals whether $\theta \in [x, z]$ or $\theta \in [z, 1]$ for some suitably chosen z satisfying $x < z < 1$—that is, the interval $[x, 1]$ is further separated into $[x, z]$ and $[z, 1]$.
    - If the meeting is deemed to be a failure, then there is no further information conveyed—that is, a babbling equilibrium in the interval $[x, 1]$ is played.

Define

$$\gamma(N) = \frac{1}{2(N+1)^2}. \tag{7}$$

Observe that $\gamma(1) = \frac{1}{8}$, $\gamma(N)$ is monotonically decreasing and $\lim_{N \to \infty} \gamma(N) = 0$. Using (4) and (7) it is easy to verify that for all $N \geqslant 2$

$$\gamma(N) < \beta(N) < \gamma(N-1) < \beta(N-1).$$

[5] Precisely, for two states θ and θ' such that $\theta < \theta'$, the distribution of equilibrium actions induced in state θ' first order stochastically dominates the distribution of equilibrium actions induced in state θ.

In fact, for all N

$$\frac{1}{\gamma(N)} = \frac{1}{2}\left(\frac{1}{\beta(N)} + \frac{1}{\beta(N+1)}\right)$$

so that $\gamma(N)$ is the harmonic mean of $\beta(N)$ and $\beta(N+1)$.

Let $b < \frac{1}{8} = \gamma(1)$. Then there exists a unique N such that

$$\gamma(N) \leqslant b < \gamma(N-1).$$

Fix such an N. Notice that if $\beta(N) \leqslant b < \gamma(N-1)$, then $N(b) = N$; whereas if $\gamma(N) \leqslant b < \beta(N)$, then $N(b) = N+1$.

Choose any $x > 0$ satisfying

$$\frac{N-1}{N+1} - 4b(N-1) < x < a_{N-1}. \tag{8}$$

We will show that for any such x, the strategies outlined above constitute an equilibrium.

The equilibrium strategies will call on the expert to distinguish states $\theta < x$ from states $\theta \geqslant x$ in the face-to-face meeting.

Play in the interval $[0, x)$: In the course of the face-to-face meeting, if the expert reveals that $\theta \in [0, x)$, then a partition equilibrium of size $N-1$ is played in the subgame that follows.

Specifically, let $z_0, z_1, z_2, \ldots, z_{N-2}, z_{N-1}$ be a partition of $[0, x)$ of size $N-1$ such that $z_0 = 0$, $z_{N-1} = x$, and for $j = 1, 2, \ldots, N-2$

$$(z_j + b) - \frac{z_{j-1} + z_j}{2} = \frac{z_j + z_{j+1}}{2} - (z_j + b),$$
$$z_{j+1} = 2z_j - z_{j-1} + 4b.$$

As in Crawford and Sobel, this second-order difference equation has a solution parametrized by z_1 (given that $z_0 = 0$):

$$z_j = z_1 j + 2j(j-1)b.$$

Then setting $z_{N-1} = x$ we obtain

$$x = z_1(N-1) + 2(N-1)(N-2)b$$

and by solving for z_1 and substituting into the difference equation above, we obtain

$$z_j = \frac{j}{N-1}x + 2bj(j-(N-1)). \tag{9}$$

Play in the interval $[x, 1]$: On the other hand, if in the course of the face-to-face meeting, the expert reveals that $\theta \in [x, 1]$, then the subsequent play depends on the second component of the conversation in the meeting, that is, on A_1 and A_2. If $S(A_1, A_2) = 1$, then in the written report the expert will further reveal whether $\theta \in [x, z]$ or $\theta \in [z, 1]$. On the other hand, if $S(A_1, A_2) = 0$, then the written report will contain no additional information, that is, it will only repeat that $\theta \in [x, 1]$.

Let z be defined by

$$(z+b) - \frac{z+x}{2} = \frac{z+1}{2} - (z+b).$$

This means that at $\theta = z$, the expert is indifferent between revealing that $\theta \in [x, z]$ and revealing that $\theta \in [z, 1]$. Then

$$z = -2b + \tfrac{1}{2}x + \tfrac{1}{2}.$$

Thus, we have shown that conditional on having initially reported honestly that the state is above or below x, the expert's incentive compatibility conditions in the continuation game hold. It remains to show that there exists a $p : 1-p$ lottery between actions $\frac{x+z}{2}$ and $\frac{1+x}{2}$ that leaves the expert indifferent to action $\frac{x+z_{N-2}}{2}$ in state x. The existence of such a lottery follows as a consequence of the next two lemmas.

Lemma 1. *At x the expert strictly prefers the action $\frac{x+z}{2}$ to $\frac{x+z_{N-2}}{2}$.*

Proof. This is the same as requiring that

$$(x+b) - \frac{x+z_{N-2}}{2} > \frac{x+z}{2} - (x+b)$$

which is equivalent to

$$x > \frac{N-1}{N+1} - 4b(N-1)$$

and this holds by our choice of x. □

Lemma 2. *At x the action $\frac{x+z_{N-2}}{2}$ induced in the interval $[z_{N-2}, x]$ is strictly preferred to the "babbling" action $\frac{x+1}{2}$ induced in the interval $[x, 1]$.*

Proof. It is sufficient to verify that

$$x + b - \frac{x+z_{N-2}}{2} < \frac{1+x}{2} - (x+b)$$

and this is equivalent to

$$x < a_{N-1}$$

which holds by our choice of x. □

We have thus shown

Proposition 1 (Existence of monotonic equilibria). *For all $b < \frac{1}{4}$, there is a continuum of perfect Bayesian equilibria of the game with face-to-face communication. The equilibrium outcomes are distinct from those of the equilibria of the CS model.*

4.2. Pareto superior equilibria

We now argue that for all $b<\frac{1}{8}$, there are equilibria of the type constructed in the previous subsection that are Pareto superior to all equilibria of the CS model. We do this by choosing a specific value of x, satisfying (8), that maximizes the decision maker's payoff among all equilibria from this class.

Before proceeding, it is useful to note the following fact, first observed by Crawford and Sobel, about the relationship between the decision maker's and the expert's ex ante expected payoffs. If we let W denote the ex ante expected payoff of the decision maker and V denote the ex ante expected payoff of the expert, then one can straightforwardly show that:

Fact 1. $V = W - b^2$.

Fact 1 implies that to show a Pareto ranking between equilibria, we need only rank them from the perspective of the decision maker.

For any x that satisfies (8), the contribution to the decision maker's equilibrium payoff from the interval $[0, x]$ is

$$\begin{aligned} W_0 &= -\sum_{j=1}^{N-1} \int_{z_{j-1}}^{z_j} \left(\frac{z_{j-1} + z_j}{2} - \theta \right)^2 d\theta \\ &= -\frac{1}{12} \frac{x^3 + 4N(N-1)^2(N-2)xb^2}{(N-1)^2}, \end{aligned}$$

where z_j are defined in (9).

The contribution to the decision maker's expected payoff from the interval $[x, 1]$ is the weighted average of two events. With probability p, the expert's written report distinguishes states $\theta \in [x, z]$ from states $\theta \in [z, 1]$ where $z = -2b + \frac{1}{2}x + \frac{1}{2}$. With probability $1 - p$, no additional information is offered. The value of p that keeps the expert indifferent is

$$p = \frac{4}{3} \frac{(N-1)^2(4N(N-2)b^2 + 4b - (1-x)^2) + x^2}{(N-1)^2(16b^2 - (1-x)^2)}. \tag{10}$$

The expected payoff in the interval $[x, 1]$ is

$$\begin{aligned} W_1 &= -p\left(\int_x^z \left(\frac{x+z}{2} - \theta \right)^2 d\theta + \int_z^1 \left(\frac{1+z}{2} - \theta \right)^2 d\theta \right) \\ &\quad - (1-p) \int_x^1 \left(\frac{1+x}{2} - \theta \right)^2 d\theta \\ &= -\frac{1}{12} \frac{4N(N-2)(N-1)^2(1-x)b^2 + 4(N-1)^2(1-x)b + x^2(1-x)}{(N-1)^2}, \end{aligned}$$

where p is defined in (10).

The overall expected payoff is

$$W(b,x) = W_0 + W_1$$
$$= -\frac{1}{12}\frac{4N(N-1)^2(N-2)b^2 + 4(N-1)^2(1-x)b + x^2}{(N-1)^2}$$

which is maximized by choosing $x^* = 2b(N-1)^2$ and this satisfies (8). The maximized value of the decision maker's equilibrium payoff for this value of x is

$$W(b) = -\tfrac{1}{3}b(1-b). \tag{11}$$

Now recall that there exists a unique N such that $\gamma(N) \leqslant b < \gamma(N-1)$. If $\beta(N) \leqslant b < \gamma(N-1)$, then $N(b) = N$ and using (6), we have

$$W(b) - U(b) = \frac{1}{12}\frac{(2bN^2-1)^2}{N^2} > 0$$

since $b < \gamma(N-1) = \frac{1}{2N^2}$. On the other hand, if $\gamma(N) \leqslant b < \beta(N)$, then $N(b) = N+1$ and

$$W(b) - U(b) = \frac{1}{12}\frac{(2b(N+1)^2-1)^2}{(N+1)^2} \geqslant 0$$

and the inequality is strict as long as $\gamma(N) = \frac{1}{2(N+1)^2} < b$.

We have thus demonstrated that there is a payoff improvement to the decision maker except at points $b = \frac{1}{2(N+1)^2}$. From Fact 1, this implies that there is a payoff improvement for the expert as well. This completes the proof of Theorem 1.

Fig. 3 displays the decision maker's expected payoff under the payoff maximizing equilibrium in the class identified in Proposition 1, denoted by $W(b)$, compared to the payoff from the best equilibrium in the CS model, denoted by $U(b)$. Notice that for almost all values of b, $W(b) > U(b)$. As noted above, the exceptions occur at points where $b = \gamma(N)$. Notice that $\gamma(1) = \frac{1}{8}$.

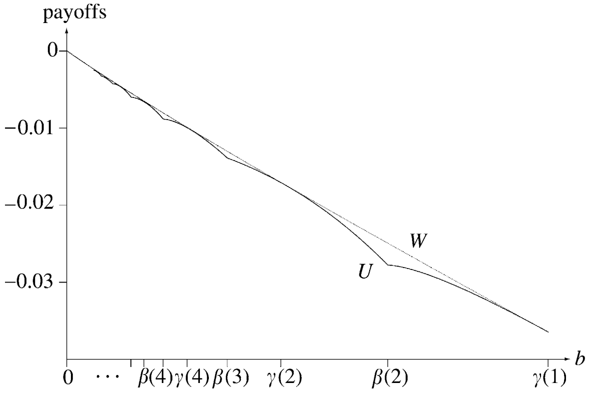

Fig. 3. Comparison of payoffs in the two models.

4.3. The role of risk-aversion

The results of the previous sections point to the benefits of face-to-face communication, benefits that accrue even if one of the parties is uninformed. The purpose of the face-to-face communication is only to introduce some randomness in how subsequent play will unfold—with some probability the subsequent play will be quite informative and with the remaining probability it will be uninformative. We have argued that on average this randomness leads to a payoff improvement for both the parties. Since both parties are risk-averse, however, at first glance this seems somewhat paradoxical—how can the introduction of additional uncertainty help risk-averse agents? But as we argue now, it is precisely the risk-aversion of both parties that leads to a payoff improvement.[6]

To isolate the effects of risk-aversion consider the following class of preferences that generalize the quadratic payoff functions considered in earlier sections. Suppose that the decision maker's utility function is of the form

$$U(y,\theta) = -|y-\theta|^{\rho},$$

whereas that of the expert is of the form

$$V(y,\theta,b) = -|y-(\theta+b)|^{\rho},$$

where $\rho \geqslant 1$ is a parameter and, as usual, b is a measure of the bias. When $\rho = 2$, this is equivalent to supposing that utility functions are quadratic.

Given a particular θ, $-|y-\theta|^{\rho}$ is a concave function of y, and it is useful to think of ρ as a measure of risk-aversion, even though the utility function is not increasing in y. In particular,

$$\rho - 1 = \left|\frac{(y-\theta)U_{yy}}{U_y}\right|$$

measures the degree of concavity of $U(\cdot,\theta)$ in a manner analogous to the Arrow–Pratt measure of relative risk-aversion.

This class of preferences has the following useful features. First, notice that equilibria in the CS model are invariant to the parameter ρ—any partition equilibrium obtained for the case $\rho = 2$ remains a partition equilibrium for all $\rho \geqslant 1$ and vice versa.

What about equilibria with face-to-face communication? Because of the randomness inherent in such equilibria, these are *not* invariant to the degree of risk-aversion. Suppose $\frac{1}{12} \leqslant b < \frac{1}{8}$. This implies that the most informative CS equilibrium involves a partition of size 2 in which the expert communicates whether the state $\theta \in [0,a_1]$ or whether $\theta \in [a_1,1]$ where $a_1 = \frac{1}{2} - 2b$. Equilibria with face-to-face communication are constructed by first choosing an $x < a_1$ and defining $z = \frac{1}{2} + \frac{1}{2}x - 2b$ so that, if it is common knowledge that $\theta \in [x,1]$, then there is an informative partition equilibrium in which, having revealed that $\theta \in [x,1]$, the expert

[6] We were led to investigate the effects of risk-aversion as a result of a question posed by Wouter Dessein.

further reveals that $\theta \in [x, z]$ or $\theta \in [z, 1]$. There is, of course, also a babbling equilibrium in which, having revealed that $\theta \in [x, 1]$, no further information is revealed. Neither of these is, of course, affected by the parameter ρ. The construction is completed by noting that in state x, the expert prefers the action $y_L = \frac{x}{2}$, optimal for the decision maker in the interval $[0, x]$, to the "pooling" action $y_P = \frac{1+x}{2}$, optimal in $[x, 1]$, but prefers the action $y_M = \frac{x+z}{2}$, optimal in $[x, z]$, to $y_L = \frac{x}{2}$ (for an example, see Fig. 1). Thus, there exists a probability p such that in state x, the expert is indifferent between revealing that $\theta \in [0, x]$ and $\theta \in [x, 1]$; that is

$$V(y_L, x, b) = pV(y_M, x, b) + (1 - p)V(y_P, x, b).$$

While the actions y_L, y_M and y_P are unaffected by changes in ρ the probability p is affected. In particular, as ρ increases p increases. This is because, in order to keep a *more* risk-averse expert indifferent between y_L and a $p : 1 - p$ lottery between the better action y_M and the worse action y_P, it is necessary to put *more* weight on the better action y_M.

We now examine how changes in ρ affect the equilibrium payoffs in our construction. The ex ante expected payoff of the decision maker in the constructed equilibrium, parameterized by x, is

$$W(x) = \int_0^x U(y_L, \theta)\, d\theta + p\left(\int_x^z U(y_M, \theta)\, d\theta + \int_z^1 U(y_H, \theta)\, d\theta\right) + (1 - p)\int_x^1 U(y_P, \theta)\, d\theta.$$

With quadratic preferences—that is, when $\rho = 2$,

$$W(x) = \tfrac{1}{3}xb - \tfrac{1}{3}b - \tfrac{1}{12}x^2.$$

When $x = a_1$, then $p = 0$, and the equilibrium is equivalent to the most informative CS equilibrium. But

$$W'(a_1) = \tfrac{2}{3}(b - \tfrac{1}{8}) < 0$$

since $b < \frac{1}{8}$. This means that there exists an open set of x's satisfying $x < a_1$ such that the corresponding equilibria with face-to-face communication all increase the expected payoffs of the decision maker relative to the most informative CS equilibrium. (In fact, W is a concave function of x and the optimal $x = 2b < a_1$.) Fact 1 implies that this open set of x's also increase the expected payoffs of the expert; hence, the equilibria under face-to-face communication are all Pareto superior.

With "absolute value" preferences—that is, when $\rho = 1$, however,

$$W(x) = -\tfrac{4}{3}b^2 - \tfrac{1}{3}bx - \tfrac{1}{3}x^2 + \tfrac{1}{4}x - \tfrac{1}{6}$$

and now

$$W'(a_1) = b - \tfrac{1}{12} > 0$$

since $b > \frac{1}{12}$. Since W is concave, this means that for all $x < a_1$, there do not exist equilibria with face-to-face communication, of the kind constructed in Proposition 1, that are Pareto superior to all equilibria in the CS model. In general, for any b, there is a $\rho^* \in (1,2)$ such that if the risk-aversion exceeds ρ^*, then there are Pareto superior equilibria.

Intuitively, the role that risk-aversion plays in improving welfare may be thought of as follows: By slightly shifting x to the left of the cutpoint in the CS model, a_1, a conversation that proves unsuccessful actually leads to a slightly Pareto worse equilibrium.[7] At the same time, this leftward shift creates the possibility of a successful conversation. In the event the conversation is successful, the equilibrium is Pareto superior to any equilibrium of the CS model. The more risk-averse is the expert, the greater the gain in the probability that a conversation will be successful from a small shift leftward in x.

For welfare to improve, this gain in probability must be sufficient to offset both the loss in welfare in the event of an unsuccessful conversation and, keeping all else fixed, the increased aversion to risk. When the expert is sufficiently risk-averse the gains outweigh these two effects.[8]

5. Extreme bias $(b > \frac{1}{8})$

In this section, we generalize the construction in Example 2 to show that when an expert's bias is extreme $(b > \frac{1}{8})$ there exist equilibria that are Pareto superior to all equilibria in the CS model. The main result of this section is:

Theorem 2. *For all $b \in (\frac{1}{8}, \frac{1}{\sqrt{8}})$, there exists a nonmonotonic perfect Bayesian equilibrium in the model with face-to-face communication which is Pareto superior to all equilibria in the CS model.*

To prove the result, we first characterize a class of nonmonotonic equilibria arising with face-to-face conversations (Section 5.1) and then show that for all values of b, an equilibrium in this class is an improvement over all equilibria in the CS model (Section 5.2).

It is useful to note that unlike all equilibria in the CS model and the construction given in Proposition 1, the equilibria constructed in this section are *nonmonotonic*—that is, they have the property that higher states are sometimes associated with (strictly) lower actions. In Section 5.3, we establish the necessity that an equilibrium be nonmonotonic for it to be beneficial compared to the most informative CS equilibrium when the expert's bias is extreme.

[7] The partition $\{[0,x],[x,1]\}$ is Pareto worse than $\{[0,a_1],[a_1,1]\}$.

[8] This is not to say that one cannot construct an equilibrium with conversations that is welfare improving when agents have absolute value preferences, only that constructions along the lines of Proposition 1 cannot lead to improvement. Indeed, one can show that more complicated constructions may improve welfare.

5.1. Construction of nonmonotonic equilibria

We construct a class of nonmonotonic equilibria along the lines of Example 2. The nature of these equilibria, whose detailed construction is provided below, is as follows.

- In the face-to-face meeting, the expert's message reveals whether the state θ lies in a set of the form $[x, z]$ or not (where $0<x<z<1$) and a second message A_1 chosen from some suitable set of messages $\mathscr{A}$. Formally, the expert's message is either of the form (In, A_1) or of the form (Out, A_1). The first component, *In* or *Out*, conveys that $\theta \in [x, z]$ or $\theta \in [0, x] \cup [z, 1]$, respectively. The decision maker also sends a message $A_2 \in \mathscr{A}$.
- Subsequent play depends on the messages that were exchanged in the face-to-face meeting.
  - If the expert says *In*, this is interpreted to mean that $\theta \in [x, z]$. In the subsequent game, the babbling equilibrium in the interval $[x, z]$ is played.
  - If the expert says *Out*, this is interpreted to mean that $\theta \in [0, x] \cup [z, 1]$. The subsequent play then depends on whether the meeting is deemed to be a success. The success or failure of the meeting is determined by a "success function" $S : \mathscr{A} \times \mathscr{A} \to \{0, 1\}$. A meeting is a success if and only if $S(A_1, A_2) = 1$. The message sets $\mathscr{A}$ and the success function S are chosen so that a particular jointly controlled lottery to be chosen below can be played.
    - If the meeting is deemed to be a success, then in the subsequent play the expert further reveals whether $\theta \in [0, x]$ or $\theta \in [z, 1]$.
    - If the meeting is deemed to be a failure, then there is no further information conveyed—that is, a babbling equilibrium in the set $[0, x] \cup [z, 1]$ is played.

We will show that for all $b \in (\frac{1}{8}, \frac{1}{\sqrt{8}})$ there exist a continuum of points x and z satisfying $0<x<z<1$ and a probability p such that the strategies outlined above constitute an equilibrium. It is convenient to refer to this kind of equilibrium as an "inside–outside" equilibrium—in the first stage the expert reveals whether the state is inside $[x, z]$ or outside.

Let

$$z = 1 - \alpha x \tag{12}$$

and notice that this entails no loss of generality. We will show that there exists an x, an $\alpha \in (0, 1)$, and a p that comprise an inside–outside equilibrium.

First, we calculate the "pooling" action y that is optimal for the decision maker when the state is revealed to be in $[0, x] \cup [z, 1]$. This is

$$\arg\min_{y'} \int_0^x (y' - \theta)^2 \, d\theta + \int_z^1 (y' - \theta)^2 \, d\theta.$$

It is easy to verify that

$$y = \frac{\alpha}{1+\alpha} + \frac{1}{2}(1-\alpha)x \tag{13}$$

and note that since $\alpha < 1$,

$$y = \frac{\alpha}{1+\alpha} + \frac{1}{2}(1-\alpha)x < \frac{1}{2} + \frac{1}{2}(1-\alpha)x = \frac{x+z}{2}.$$

In other words, the "pooling" action, y, that is optimal in the set $[0,x] \cup [z,1]$ is *lower* than the "medium" action $\frac{x+z}{2}$ that is optimal in the middle interval $[x,z]$.

The points x and z and the probability p must be such that when the state is x, the expert is indifferent between a $p:1-p$ lottery over actions $\frac{x}{2}$ and y on the one hand, and the action $\frac{x+z}{2}$ on the other. Similarly, when the state is z, the expert is indifferent between a $p:1-p$ lottery over actions $\frac{1+z}{2}$ and y on the one hand, and the action $\frac{x+z}{2}$ on the other. The two "no arbitrage" conditions are:

$$(1-p)(y-(x+b))^2 + p\left(\frac{x}{2}-(x+b)\right)^2 = \left(\frac{x+z}{2}-(x+b)\right)^2,$$

$$(1-p)(y-(z+b))^2 + p\left(\frac{1+z}{2}-(z+b)\right)^2 = \left(\frac{x+z}{2}-(z+b)\right)^2.$$

The (x,p) pair that solves this system of simultaneous equations is

$$x(\alpha) = \frac{(1+\alpha)(4b(1-\alpha)+5(1+\alpha)) - \sqrt{\phi(\alpha,b)}}{4(1+\alpha)(\alpha^2+4\alpha+1)}, \tag{14}$$

where

$$\phi(\alpha,b) = (1+\alpha)[48(3+\alpha)(1+3\alpha)(1+\alpha)b^2 + 8(1-\alpha)(9\alpha^2+26\alpha+9)b + (1+\alpha)(9\alpha^2-14\alpha+9)]$$

and

$$p(\alpha) = \frac{(1-4b) - 3\alpha^2 + 4b\alpha^2 - 2x + 2\alpha(1-x) + 2\alpha^3 x + 2\alpha^2 x}{(2-x-\alpha x)\alpha(\alpha^2 x + 3\alpha x + 2x + 4b(1+\alpha) - 2\alpha)}, \tag{15}$$

where we have suppressed the dependence of x and p on the bias parameter b.

We next show that there exists a value of α close to $\alpha = 1$ such that the point x and the probability p, as determined in (14) and (15), respectively, together with $z = 1 - \alpha x$, comprise an equilibrium.

Notice that the unique value of α where $x(\alpha) = 0$ is $\alpha^* = \frac{2b-(1-8b^2)}{2b+1-8b^2}$. It is easy to verify that $\alpha^* < 1$ if and only if $b < \frac{1}{\sqrt{8}}$.

We first check that when α is close to 1, the resulting solution for $x(\alpha)$ and $z(\alpha)$ is feasible. When $\alpha = 1$, $x = \frac{5}{12} - \frac{1}{12}\sqrt{1+192b^2}$. Notice that as long as $b < \frac{1}{\sqrt{8}}$, this value of x is strictly positive and less than $\frac{1}{2}$. Hence, values of α close to 1 yield values of x between 0 and 1. Further, since $x < \frac{1}{2}$, then $z = 1 - \alpha x > x$; hence this yields a feasible value of z as well.

We next check that when α is close to 1, the solution for p is feasible, that is $p \in (0,1)$. Notice that when $\alpha = 1$, $p(1) = 0$. Moreover, this is the only $\alpha \in [0,1]$ where $p = 0$. Further, $\lim_{\alpha \to 0} p(\alpha) = \frac{4}{3(4b+3)} > 0$ (using L'Hôspital's rule to evaluate the limit of $p(\alpha)$ from (15)). Hence, for values of α close to 1 the solution for p is feasible.

Now consider the function $f(\theta)$, which is just the difference between the expected payoff of the expert in state θ from the $p : 1-p$ lottery of actions $\frac{x}{2}$ and y and the payoff in state θ from the action $\frac{x+z}{2}$.

$$f(\theta) = (1-p)(y - (\theta+b))^2 + p\left(\frac{x}{2} - (\theta+b)\right)^2 - \left(\frac{x+z}{2} - (\theta+b)\right)^2 \tag{16}$$

and recall that by construction, $f(x) = 0$. Now

$$\begin{aligned} f'(\theta) &= -2(1-p)(y-(\theta+b)) - 2p\left(\frac{x}{2} - (\theta+b)\right) + 2\left(\frac{x+z}{2} - (\theta+b)\right) \\ &= -2(1-p)y - 2p\frac{x}{2} + 2\frac{x+z}{2} \\ &= 2\left(\frac{x+z}{2} - \left(p\frac{x}{2} + (1-p)y\right)\right) \\ &> 0 \end{aligned}$$

since $y < \frac{x+z}{2}$. So for all $\theta < x$, $f(\theta) < 0$ and for all $\theta > x$, $f(\theta) > 0$.

Similarly, consider the function $g(\theta)$, which is the difference between the expected payoff of the expert in state θ from the $p : 1-p$ lottery of actions $\frac{1+z}{2}$ and y and the payoff in state θ from the action $\frac{x+z}{2}$.

$$g(\theta) = (1-p)(y-(\theta+b))^2 + p\left(\frac{1+z}{2} - (\theta+b)\right)^2 - \left(\frac{x+z}{2} - (\theta+b)\right)^2 \tag{17}$$

and, as before, by construction, $g(z) = 0$. Now

$$\begin{aligned} g'(\theta) &= -2(1-p)y - 2p\frac{1+z}{2} + 2\frac{x+z}{2} \\ &= 2\left(\frac{x+z}{2} - \left(p\frac{1+z}{2} + (1-p)y\right)\right). \end{aligned}$$

But notice that $\lim_{\alpha \to 1} g'(\theta) < 0$ since $y(1) = \frac{x(1)+z(1)}{2}$. Thus, for α close to 1, it is also the case that $g'(\theta) < 0$. This implies that for all $\theta < z$, $g(\theta) > 0$ and for all $\theta > z$, $g(\theta) < 0$. Together, the conditions on f and g imply that it is incentive compatible for the expert to reveal whether or not the state is in $[x, z]$ in the first stage.

It remains to verify that once it has been revealed in the first stage that $\theta \in [0,x] \cup [z,1]$ and the face-to-face meeting has been a success, then it is the case that the expert is willing to separate $[0,x]$ from $[z,1]$. But this also follows from the facts derived above. For instance, if $\theta \in [0,x]$ then $f(\theta) < 0$ and $g(\theta) > 0$ together imply that

$$\left(\frac{x}{2} - (\theta+b)\right)^2 < \left(\frac{1+z}{2} - (\theta+b)\right)^2$$

so that the expert is willing to separate $[0,x]$ from $[z,1]$. Similarly, when $\theta > z$, the opposite inequality holds.

Thus, for all α close to 1, there is an inside–outside equilibrium of the kind described above. We have shown

Proposition 2 (Existence of nonmonotonic equilibria). *For* $b \in [\frac{1}{8}, \frac{1}{\sqrt{8}})$, *there is a continuum of nonmonotonic perfect Bayesian equilibria of the game with face-to-face communication. The equilibrium outcomes are distinct from those of the equilibria of the CS model.*

5.2. Pareto superior equilibria

Proposition 2 shows that for all α close to 1, there exist nonmonotonic "inside–outside" equilibria. For values of $b \geqslant \frac{1}{4}$, such equilibria constructed are clearly more informative than the CS equilibrium, which entails no information transmission whatsoever on the part of the expert. This implies that the expected payoffs to the decision maker are higher under an inside–outside equilibrium than under a babbling equilibrium, and, because of Fact 1, this implies that inside–outside equilibria are Pareto superior.

For values of $b < \frac{1}{4}$, however, inside–outside equilibria when α is close to 1 do not improve payoffs relative to the "best" CS equilibrium. To establish Theorem 2, we still need to show that Pareto superior equilibria can also be constructed for values of $b \in (\frac{1}{8}, \frac{1}{4})$. We do this by considering inside–outside equilibria of the kind described above for values of α which are close to 0. Specifically, we show that, for all $b \in (\frac{1}{8}, \frac{1}{4})$, there exists an α close to 0 that results in a Pareto superior equilibrium.

Suppose $b \in (\frac{1}{8}, \frac{1}{4})$. Recall that for these values of b, the most informative CS equilibrium is one with a two-element partition which breaks the state space into two at the point $\frac{1}{2} - 2b$. Now notice that as $\alpha \to 0$, then from (14) $x(0) = \frac{1}{2} - 2b$. Moreover, in that case $z = 1$. Thus when $\alpha = 0$, the inside–outside equilibrium reduces to the most informative CS equilibrium.[9]

To verify that there is an inside–outside equilibrium for values of α close to 0, we look at the limiting properties of the relevant variables. We know that $\lim_{\alpha \to 0} x(\alpha) = x(0) = \frac{1}{2} - 2b \in (0, \frac{1}{2})$. It may be verified that

$$\lim_{\alpha \to 0} p(\alpha) = \frac{8}{3(4b+3)}$$

(by using L'Hôspital's rule in (15)), so that $\lim_{\alpha \to 0} p(\alpha) \in (0, 1)$. Since $\lim_{\alpha \to 0} x(\alpha)$ and $\lim_{\alpha \to 0} p(\alpha)$ are both strictly between 0 and 1, it is the case that for values of α close to 0, $x(\alpha)$ and $p(\alpha)$ are also feasible. Moreover, $x(\alpha) < \frac{1}{2}$, so that $x(\alpha) < z(\alpha) = 1 - \alpha x(\alpha)$.

[9] Notice that $x(0)$ is infeasible if $b \geqslant \frac{1}{4}$. Thus there are inside–outside equilibria for values of α close to 0 only if $b < \frac{1}{4}$. In contrast, for values of α close to 1 there are inside–outside equilibria for all b between $\frac{1}{8}$ and $\frac{1}{\sqrt{8}}$.

Now consider the functions f and g as defined in (16) and (17), respectively. As before, for all $\theta < x, f(\theta) < 0$ and for all $\theta > x, f(\theta) > 0$ (the argument in the previous subsection did not rely on the particular value of α). But recall that

$$g'(\theta) = 2\left(\frac{x+z}{2} - \left(p\frac{1+z}{2} + (1-p)y\right)\right)$$

and it may be verified that

$$\lim_{\alpha \to 0} g'(\theta) = -\tfrac{1}{3} < 0.$$

Thus, when α is close to zero, for all $\theta < z$, $g(\theta) > 0$ and for all $\theta > z$, $g(\theta) < 0$. As above, this completes the proof that x and p comprise an equilibrium.

It remains to argue that for $b \in (\frac{1}{8}, \frac{1}{4})$ and α is close to 0, the constructed equilibrium is Pareto superior to the most informative CS equilibrium. Again, relying on Fact 1, it is sufficient to show that the decision maker earns higher expected payoffs under the nonmonotonic equilibrium than under the best CS equilibrium.

For arbitrary $\alpha \in (0,1)$, the expected utility of the decision maker in an inside–outside equilibrium is

$$W(\alpha) = -\int_0^x \left(p\left(\frac{x}{2} - \theta\right)^2 + (1-p)(y-\theta)^2\right) d\theta - \int_x^z \left(\frac{x+z}{2} - \theta\right)^2 d\theta \\ - \int_z^1 \left(p\left(\frac{1+z}{2} - \theta\right)^2 + (1-p)(y-\theta)^2\right) d\theta.$$

Recall that $W(0)$ is the same as the decision maker's expected utility in the best CS equilibrium; that is,

$$W(0) = -\frac{1}{12N(b)^2} - \frac{b^2(N(b)^2-1)}{3},$$

where $N(b) = 2$ since $b \in (\frac{1}{8}, \frac{1}{4})$.

Now it can be shown that

$$\lim_{\alpha \to 0} W'(\alpha) = \tfrac{1}{3} b(8b-1) > 0$$

for all $b \in (\frac{1}{8}, \frac{1}{4})$. (The details of this computation may be obtained from the authors.)

Since $W(\alpha)$ is equal to the payoff from the best CS equilibrium when $\alpha = 0$ and $\lim_{\alpha \to 0} W'(\alpha) > 0$, for all $b \in (\frac{1}{8}, \frac{1}{4})$ we conclude that for all such b, when α is close to 0, there is an inside–outside equilibrium that is Pareto superior to the best CS equilibrium.

Fig. 4 summarizes the key features of inside–outside equilibria. Inside–outside equilibria are feasible whenever α lies above the lower of the two curves; in particular, they exist for values of α close to 1—this is the content of Proposition 2. Inside–outside equilibria are Pareto superior to any CS equilibria whenever α lies below the higher of the two curves. For $b \geqslant \frac{1}{4}$, every inside–outside equilibrium is more informative than CS equilibria since the latter are completely uninformative.

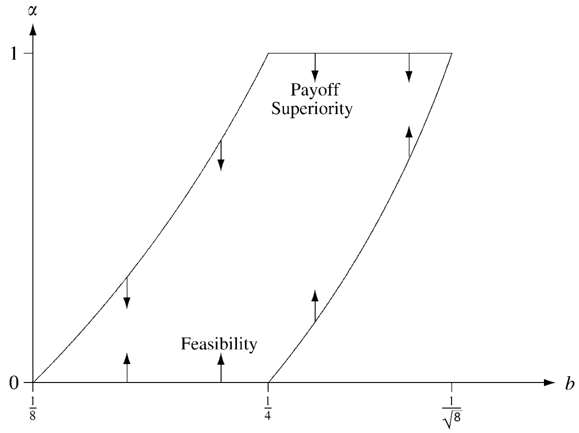

Fig. 4. Feasible and payoff superior inside–outside equilibria.

For $b \in (\frac{1}{8}, \frac{1}{4})$, inside–outside equilibria are Pareto superior for values of α close to 0, as established in the argument of this section.

5.3. Monotonic equilibria

We end this section by showing that nonmonotonic equilibria are in fact *necessary* to achieve Pareto gains when the expert's bias is extreme. Specifically, we show that for $b \geqslant \frac{1}{8}$, there cannot be a monotonic equilibrium that improves the outcome compared to the most informative CS equilibrium. For values of $b \in [\frac{1}{8}, \frac{1}{4})$ monotonic equilibria do exist, but they are Pareto dominated by the most informative CS equilibrium. For values of $b \geqslant \frac{1}{4}$, the only monotonic equilibrium involves babbling as in Crawford and Sobel.

As a first step, we show that all monotonic equilibria are partitional. That is, the expert reveals a partition of the state space which consists of intervals in the initial conversation, and there is (possibly) further partitioning of the state space in the written report stage of the communication between the two parties.

Lemma 3. *In any monotonic equilibrium of the game with one stage of conversation, the information revealed in the first stage is partitional.*

Proof. Suppose we have two states θ and θ' such that $\theta < \theta'$. Let $F(\cdot|\theta)$ be the distribution of equilibrium actions in state θ etc. Monotonicity implies that for all actions $y \in [0, 1]$, $F(y|\theta') \leqslant F(y|\theta)$. Suppose further that $F(\cdot|\theta') \neq F(\cdot|\theta)$.

In state ω, the difference in the expert's expected utility from inducing the distribution of actions associated with state θ versus inducing the distribution associated with θ' is

$$\Delta(\omega) = E_Y[U(Y, \omega, b)|\theta] - E_Y[U(Y, \omega, b)|\theta'].$$

Differentiating with respect to ω, we obtain that the derivative of this difference

$$\Delta'(\omega) = \int_0^1 U_2(y, \omega, b)\, dF(y|\theta) - \int_0^1 U_2(y, \omega, b)\, dF(y|\theta').$$

Now since $U_{12} > 0$, $U_2(\cdot, \omega, b)$ is an increasing function and so the fact that $F(y|\theta')$ stochastically dominates $F(y|\theta)$ implies that $\Delta'(\omega) < 0$.

Incentive compatibility implies that

$$\Delta(\theta) \geqslant 0,$$
$$\Delta(\theta') \leqslant 0$$

with at least one strict inequality.

This means that there is a unique state $\theta^* \in [\theta, \theta']$, such that $\Delta(\theta^*) = 0$. For all $\omega < \theta^*$, $\Delta(\omega) > 0$ and for all $\omega > \theta^*$, $\Delta(\omega) < 0$. Thus, the expert will never induce the distribution associated with θ' for any states $\omega < \theta^*$. Likewise, the expert will never induce the distribution associated with θ for any states $\omega > \theta^*$. Therefore, the information revealed in the first stage conversation is partitional.

There can be only finitely many such intervals. To see this, notice that if the interval in which a given distribution of actions is induced is sufficiently small then only a babbling equilibrium is possible in the written report stage and therefore the distribution of actions associated with these states is degenerate. Further, if two such intervals occur consecutively, then Lemma 1 of Crawford and Sobel applies directly so that the induced actions must be a finite distance apart. On the other hand, an interval in which a nondegenerate distribution of actions occurs must also be sufficiently large that it admits a nonbabbling equilibrium. □

We are now in a position to establish that

Proposition 3. *For all $b \geqslant \frac{1}{8}$, all monotonic equilibria of the game with face-to-face communication are Pareto inferior to the best CS equilibrium.*

Proof. First, suppose $b \in [\frac{1}{8}, \frac{1}{4})$. From Lemma 3, we know that the information contained in the first stage conversation is partitional. Further, since, for a partition equilibrium of the written report stage to be nonbabbling, the subinterval must be of length at least $4b$, this then implies that the only feasible partition in the conversation stage divides the state space into two intervals: $[0, x]$ and $[x, 1]$. In the written report stage, either $[0, x]$ is further subdivided into $[0, z]$ and $[z, x]$ and the interval $[x, 1]$ is not subdivided any further; or $[0, x]$ is not subdivided but $[x, 1]$ is (as in the previous section).[10] We now argue that the first possibility can be ruled out.

Consider such an equilibrium and suppose that, with some probability, the interval $[0, x]$ is separated into subintervals $[0, z]$ and $[z, x]$. The separation

[10] Since $b \geqslant \frac{1}{8}$, this implies that $x > \frac{1}{2}$, so there is insufficient room to separate over the interval $[x, 1]$ when separating over $[0, x]$.

requires that

$$z = -2b + \tfrac{1}{2}x.$$

For $z > 0$ requires $x > 4b$.

Moreover, it must be that there is a p such that when the state is x, the expert is indifferent between the action $\frac{x+1}{2}$ for sure and a $p : 1-p$ lottery between $\frac{x+z}{2}$ and $\frac{z}{2}$. Thus we must have

$$p\left(\frac{x+z}{2} - (x+b)\right)^2 + (1-p)\left(\frac{x}{2} - (x+b)\right)^2 = \left(\frac{x+1}{2} - (x+b)\right)^2. \tag{18}$$

The necessary conditions for a $p \in (0,1)$ that solves Eq. (18) are

$$\frac{1+x}{2} > x+b,$$
$$\frac{1+x}{2} - (x+b) > x+b - \frac{x+z}{2}.$$

The second inequality reduces to (after substituting for z)

$$\tfrac{1}{2} > \tfrac{3}{4}x + 3b. \tag{19}$$

Since $x > 4b$, the inequality required in Eq. (19) implies that

$$\tfrac{1}{2} > 6b$$

which is a contradiction since $b \geqslant \frac{1}{8}$.

Thus, when $b \geqslant \frac{1}{8}$ all monotonic equilibria must involve separation of an interval of the form $[x, 1]$.

The expected utility from such an equilibrium is

$$\begin{aligned} W(x) = & -\int_0^x \left(\frac{x}{2} - (\theta+b)\right)^2 d\theta \\ & - p\left(\int_x^z \left(\frac{x+z}{2} - (\theta+b)\right)^2 d\theta + \int_z^1 \left(\frac{1+z}{2} - (\theta+b)\right)^2 d\theta\right) \\ & - (1-p)\int_x^1 \left(\frac{1+x}{2} - (\theta+b)\right)^2 d\theta \\ = & -\frac{1}{12}x^2 + \frac{1}{3}xb - \frac{1}{3}b - b^2. \end{aligned}$$

The x which maximizes $W(x)$ is $x^* = 2b$; however this value of x is only feasible when $x < a_1$, which it is not when $b \geqslant \frac{1}{8}$. Thus, the welfare maximizing feasible value of x is $x = a_1$, which is outcome equivalent to the best CS equilibrium.

When $b \geqslant \frac{1}{4}$, the unique monotonic equilibrium is babbling as in the CS model. □

6. Extensions

In this section, we consider two extensions of the amended model. In Section 6.1, we show that introducing a mediator into the conversation between the expert and the decision maker can lead to further informational improvement over the equilibria we identified, even absent multiple stages or active participation by the decision maker. In Section 6.2, we show that a central result of the CS model, that divergent preferences necessarily give rise to information withholding in equilibrium, continues to hold regardless of the number of stages of conversation or the active participation of the decision maker.

6.1. Mediated talk

In the preceding sections we have emphasized the benefits of *plain conversation* between the decision maker and the expert—that is, the mode of communication is what we have called "face-to-face" and does not involve the use of any outside agencies. An alternative mode is to make use of an external *mediator* (see [10,12,20]) who functions as follows: First, the expert reports a state θ to the mediator. The mediator then suggests an action y to the decision maker that is chosen at random from some known probability distribution $p(y|\theta)$. A mediator is said to be incentive compatible if (i) the decision maker has the incentive to choose the suggested action y given his posterior beliefs $q(\cdot|y)$ on the state of nature induced by the suggestion; and (ii) given the probability distribution $p(y|\theta)$, the expert has the incentive to reveal the state truthfully to the mediator.

Notice that any equilibrium of a model with plain conversation can be induced via a mediator—one that duplicates the implied probability distributions $p(\cdot|\theta)$. However, since the mediator himself is assumed to have no preferences and so the mediator's actions are not subject to any incentive compatibility constraints, he can sometimes improve information transmission over and above what is possible with plain conversation.

To illustrate the benefits of mediated talk, we consider a case of very extreme bias, say $b = \frac{1}{\sqrt{8}}$. In this case, the only equilibrium of the CS model is completely uninformative, nor is there a monotonic equilibrium of the game with face-to-face communication in which any information transmission takes place.[11] Finally, it can be argued that there is no informative inside–outside nonmonotonic equilibrium of the kind constructed in the previous section. A mediator, however, can facilitate useful information transmission even when the bias is as extreme as that considered here. We now construct an explicit example to illustrate this.

Let $x = \frac{1}{8}$ and define $K = \sqrt{7409 - 5200\sqrt{2}}$. Let

$$y_1 = \frac{1}{16}\frac{545 - 296\sqrt{2} - 9K}{73 - 40\sqrt{2} - K} \simeq 0.413,$$

$$y_2 = \frac{1}{16}\frac{575 - 344\sqrt{2} + 9K}{67 - 40\sqrt{2} + K} \simeq 0.544.$$

[11] This follows from Lemma 3.

The mediator suggests only actions $y \in \{y_1, y_2\}$. If $\theta < x$, then y_1 is suggested with probability $p_{11} = \frac{4}{5}$ and y_2 with probability $p_{12} = 1 - p_{11}$. If $\theta \geqslant x$, then y_1 is suggested with probability p_{21} and y_2 with probability $p_{22} = 1 - p_{21}$, where

$$p_{21} = \frac{2}{35} \frac{59 - 32\sqrt{2} - K}{7 - 4\sqrt{2}} \simeq 0.269.$$

Given that y_i is suggested, the posterior probability assigned by the decision maker to the event that $\theta < x$ is

$$q_{1i} = \frac{xp_{1i}}{xp_{1i} + (1-x)p_{2i}}$$

and it may be verified that for $i = 1, 2$

$$y_i = \arg\min_y \; q_{1i} \frac{1}{x} \int_0^x (y - \theta)^2 \, d\theta + (1 - q_{1i}) \frac{1}{1-x} \int_x^1 (y - \theta)^2 \, d\theta.$$

Thus, it is incentive compatible for the decision maker to follow the mediator's recommendations.

Given the randomizing device used by the mediator, the expert has no incentive other than to tell the truth. This is because in state θ, the difference in the expert's payoffs from reporting that the state is greater than x rather less than x is

$$p_{11}(y_1 - (\theta + b))^2 + p_{12}(y_2 - (\theta + b))^2 - (p_{21}(y_1 - (\theta + b))^2 + p_{22}(y_2 - (\theta + b))^2)$$

which is negative when $\theta < x$ and positive when $\theta > x$. Thus it is incentive compatible for the expert to reveal the state truthfully to the mediator.

The equilibrium constructed above is monotonic—$y_1 < y_2$ and the distribution of actions (p_{21}, p_{22}) induced when $\theta \geqslant x$ stochastically dominates the distribution of actions (p_{11}, p_{12}) induced when $\theta < x$. Thus, with mediated talk, there is an informative monotonic equilibrium even though with face-to-face communication there is none.

One can say something stronger than this: the outcome in the mediated game given above is not an equilibrium (monotonic or otherwise) of any k stage conversation. To see this, consider a one stage conversation. If, following any conversation, there were no additional information in the expert's final report, then the conversation itself is irrelevant. Thus, if that were the case, then the expert would be choosing between actions y_1 and y_2 in the first stage. However, the only states for which it is optimal to induce y_1 over y_2 are those where $\theta \leqslant x$, but knowing that $\theta \leqslant x$, it is no longer optimal for the decision maker to select y_1. Therefore, the expert must be revealing additional information in his final report. However, again this information must ultimately determine whether the decision maker selects y_1 or y_2 and the same condition holds; therefore, y_1 is again not optimal for the decision maker. Hence, this outcome is not attainable in any one stage conversation.

By backward induction, the same argument may be used to establish that this outcome is not attainable in a k stage conversation where k is finite.

6.2. Full revelation

Consider a variation of the amended game where for $k>0$ stages the expert and the decision maker meet face-to-face, followed by the expert issuing a written report, and the decision maker taking some action. We show:

Proposition 4. *All equilibria with conversations are bounded away from full revelation.*

Proof. Suppose not. Then there is a sequence of equilibrium messages m^n and probability distributions over equilibrium messages $\mu^n(\theta)$ such that for all ε, there exists an N such that for all $n \geqslant N$,

$$\mu^n(\theta)(\{m : |y(m) - \theta| > \varepsilon\}) < \varepsilon,$$

where $y(m)$ is the action chosen following message m.

Let $\theta' < \theta''$ and let these be close enough so that $\theta'' < \theta' + b$. Let

$$\varepsilon < \min\{\tfrac{1}{2}(\theta'' - \theta'), (\theta' + b) - \theta''\}.$$

This implies, in particular, that in state θ' the expert strictly prefers every y in an ε neighborhood of θ'' to every y in an ε neighborhood of θ'.

So in state θ' if the expert were to behave as if the state were θ'', then for small enough ε, the resulting distribution over actions would be preferred to the distribution if he were to behave truthfully. This is a contradiction. □

Note that Proposition 4 extends unchanged to the case of mediated talk.

7. Discussion

We have shown that a simple and natural amendment of the CS model leads to more information disclosure by an expert. Specifically, when the decision maker and expert have a face-to-face conversation prior to the issuance of the expert's final written report, more information is revealed in equilibrium than where no such conversation precedes the written report. This is the case even though the decision maker is completely uninformed.

The decision maker injects only randomness into the conversation, and ultimately into the actions taken in response to the expert's advice. By breaking the deterministic link between expert's advice and the ultimate action undertaken, the incentives for the expert to strategically withhold information are reduced. Somewhat paradoxically, the injection of uncertainty is typically more effective when the expert is more risk-averse.

Allowing for active participation by the decision maker and multiple stages of communication into the model leads to sets of equilibria that are more complex, in several dimensions, than the characterization in the CS model. First, in their model, there are only finitely many equilibria. With conversations, there are a continuum of equilibria. Second, for a given number of partition elements arising in an equilibrium in their model, there is a unique partition structure. This is not the case for the equilibria with conversations. Third, all CS equilibria are monotonic—higher actions are associated with higher states. With conversations, equilibria may be non-monotonic. Finally, and most importantly, we have shown that the predictions of the CS model about information withholding by experts are, in a sense, overly pessimistic.

References

[1] R. Aumann, S. Hart, Long cheap talk, Econometrica 71 (2003) 1619–1660.
[2] R. Aumann, M. Maschler, Repeated Games with Incomplete Information, MIT Press, Cambridge, 1995.
[3] I. Barany, Fair distribution protocols or how players replace fortune, Math. Oper. Res. 17 (1992) 327–340.
[4] D. Baron, Legislative organization with informational committees, Amer. J. Polit. Sci. 44 (2000) 485–505.
[5] M. Battaglini, Multiple referrals and multidimensional cheap talk, Econometrica 70 (2002) 1379–1401.
[6] B.D. Bernheim, S. Severinov, Bequests as signals: an explanation for the equal division puzzle, Mimeo, University of Wisconsin, 2001.
[7] V. Crawford, J. Sobel, Strategic information transmission, Econometrica 50 (1982) 1431–1451.
[8] W. Dessein, Authority and communication in organizations, Rev. Econ. Stud. 69 (2002) 811–838.
[9] J. Farrell, R. Gibbons, Cheap talk can matter in bargaining, J. Econ. Theory 48 (1989) 221–237.
[10] F. Forges, An approach to communication equilibria, Econometrica 54 (1986) 1375–1385.
[11] F. Forges, Equilibria with communication in a job market example, Quart. J. Econ. 105 (1990) 375–398.
[12] F. Forges, Universal mechanisms, Econometrica 58 (1990) 1341–1364.
[13] T. Gilligan, K. Krehbiel, Collective decision-making and standing committees: an informational rationale for restrictive amendment procedures, J. Law Econ. Org. 3 (1987) 287–335.
[14] T. Gilligan, K. Krehbiel, Asymmetric information and legislative rules with a heterogeneous committee, Amer. J. Polit. Sci. 33 (1989) 459–490.
[15] G. Grossman, E. Helpman, Special Interest Politics, MIT Press, Cambridge, 2001.
[16] V. Krishna, J. Morgan, A model of expertise, Quart. J. Econ. 116 (2001) 747–775.
[17] V. Krishna, J. Morgan, Asymmetric information and legislative rules: some amendments, Amer. Polit. Sci. Rev. 95 (2001) 435–452.
[18] S. Matthews, A. Postlewaite, Pre-play communication in a two-person sealed-bid double auction, J. Econ. Theory 48 (1989) 238–263.
[19] J. Morgan, P. Stocken, An analysis of stock recommendations, RAND J. Econ. 34 (2003) 183–203.
[20] R. Myerson, Multistage games with communication, Econometrica 54 (1986) 323–358.